Mathematics

Grade 11

Teacher's Instructional Manual

(Operational from year 2008)

Department of Mathematics
Faculty of Science and Technology
National Institute of Education
Maharagama

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Message of the Director General

The first curriculum revision for the new millennium aims to eliminate several problems that exist in the present education system. Having identified the problems that youth face due to the weakening of thinking, social and personal skills, and investigating step by step the factors that have resulted in this situation, this curriculum reform has been planned in order to create the environment that is necessary to overcome this.

Our country took the lead in education in the Asian region in the past. However many other countries in this region have now advanced ahead of us. Some of the factors that have caused this decline are the actions that have been taken continuously by the educational institutions over a period of time, to retain what is known, to learn the pre-determined and construct what is in the same manner as before.

By taking all these factors into consideration, the officers of the National Institute of Education have endeavored to compile the new curriculum with a clearer vision. The primary objective of this curriculum is to produce a generation of students who will display their readiness for a successful future, by changing what is known, exploring what is new and building up what is required for the future. It is unnecessary to reiterate that to fulfill this goal, there should be a distinct change in the teacher's role. Instead of the transmission and transactional teacher role that has been prominent in our classrooms to date, teachers in the Sri Lankan schools will have to understand and familiarize themselves with a student centered, competency based and activity focused transformational role under this new curriculum.

It is our firm belief that this Teacher's Instructional Manual will aid you to adapt yourself to the new situation and to become an effective teacher in the new millennium. By following these instructions you will be helped in your daily teaching and evaluation duties. There is no doubt that the instructions for student exploration and other quality inputs will also facilitate the teacher's task. This Teacher's Instructional Manual also provides valuable information to school principals regarding the preparation of timetables, sharing of limited resources and internal supervision.

My sincere thanks go to Dr. Mrs. I. L. Ginige, Assistant Director General (Curriculum Development), Faculty of Science & Technology of the National Institute of Education for her direct involvement in the preparation of this Teacher's Instructional Manual. My thanks are also due to the other officers and all other personnel who aided in the preparation of this manual that will serve not only the above mentioned persons involved at the school level but also the teacher educationists involved in beginning or in continuous teacher education duties, in-service advisors as well as officers at various levels who are involved in external supervision and monitoring programmes.

Professor J. W. Wickramasinghe
Director General
National Institute of Education
Maharagama

2007.10.06

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Preface

The first curriculum reforms of the new millennium which are being implemented with the objective of laying a strong foundation for a new Sri Lanka, anticipates a visible transformation in the role of the teacher. The Teacher's Instructional Manual which has been compiled to provide the support necessary to achieve this, contains the following main sections.

- Descriptive syllabus
- Activity continuum to aid in the implementation of the syllabus
- Instruments for the extension of the learning-teaching process

The descriptive syllabus which extends further than the subject topics and sub-topics, provides teachers with an understanding of some of the main issues that were considered in designing the curriculum. This section, which begins with an introduction of the basis for the new syllabus and the subject aims, includes competency levels that have been prepared in line with the subject competencies. The subject knowledge that students should gain under each competency level is introduced as subject content. A special feature of this section is that by considering the many learning-teaching methods that are used to provide students with the required knowledge, the time frame required for each competency level has been determined. It is essential that every instructional leader carefully reads and understands the final section of this descriptive syllabus which is presented under the topic "School Policies and Programmes". This section provides school managers with a range of valuable instructions to assist them in allocating time for teaching, assigning teaching duties, implementing co-curricular activities and supervising teachers' tasks.

The second section of the teacher's instructional manual has been prepared with the aim of providing teachers with a clear understanding of the proposed learning-teaching methodologies. This section commences with the desired change in the teacher's role and an introduction to the procedures that should be followed to prepare activities under a competency based education. The activity continuum of the curriculum is presented next. The teacher is not expected to carry out the activities exactly as presented. He/she should use his/her creative ability and reasoning to adapt the activities to suit the class. Although instructions have been provided based on each problem that is to be explored, on the number of groups that the class should be separated into, it is expected that the teacher will use his/her discretion to group the class wisely taking into consideration on the number of students in the class.

Time has been allocated for the activities to enable the relevant competency levels to be achieved. Thus teachers will have to go beyond the usual 40 minutes period. While each activity has been provided with adequate time for the competency level to be achieved, the teacher is expected to make use of single or double periods in the time-table to breakdown the activities in a suitable manner. Whenever an activity which has commenced on a previous day is continued on the following day, it is essential that a brief summary of what has been carried out previously is presented to the class for the success of this process. This decision will also provide the school community with the opportunity to engage students in effective learning whenever a teacher is absent. The final item in this section is the list of quality inputs that are necessary to maintain the quality of the learning-teaching process taken as a whole. Provision of this list enables the teacher to order the necessary quality inputs in time and have them in hand when required.

A number of important hints to ensure that the results expected from this syllabus are achieved are included in the third section of the teacher's instructional manual under the title "Assessment and Evaluation". This section has been planned to introduce the assessment and evaluation that should take place under each activity, instruments for the extension of the learning teaching process and the nature of the questions that could be expected at general examinations. The main responsibility of the teacher is to identify when assessment and evaluation can take place in the course of each activity, and to carry it out successfully through a common set of criteria. The set of instruments for the extension of the learning teaching process provide students with the opportunity to be involved in continuous learning outside the recommended classroom sessions. It is the responsibility of the teacher to encourage the students and regularly assess the competencies developed by them through these instruments. It is also expected that the teacher will make correct judgments regarding the final outcomes of the activities and communicate them to the relevant parties.

There should be a distinct change in the questions in the general examinations for the success of this learning teaching process. The National Institute of Education with the assistance of the Department of Examinations has worked towards introducing several prototype questions to the educational levels that terminate with a general examination. Since this change in the examination questions has been suggested in order to direct students to learn through activities and experiences instead of through mechanical approaches such as memorizing and answering model papers, students and parents should be informed of this change at the initial stage itself.

All teachers should understand that various activities can be developed to achieve the required competency levels. They should be prepared therefore for more innovative teaching through better approaches, exploration and instruments for the extension of the learning-teaching process, instead of implementing the proposed activities exactly as instructed. This teacher's instructional manual will give teachers throughout the country the courage to effect a visible change in the teacher's role and to prevent them from becoming inactive in the face of new approaches. We expect to award certificates and provide numerous development opportunities to encourage teachers who go beyond the given activities and involve themselves in novel creations. To be eligible for these awards, the teachers should use their creative abilities to further improve the proposed activities. These should be forwarded to the Assistant Director General (Curriculum Development), Faculty of Science and Technology, National Institute of Education, Maharagama. The activities will be examined by the relevant subject committees before a final decision is made regarding the awards.

We have endeavored through this effort to bring learning, teaching, assessment and evaluation onto the same platform through new methodologies. By this, the teacher is provided with latitude to carry out the learning – teaching process, school based evaluation and homework assignments in a meaningful manner. It is our firm conviction that the Sri Lankan school system will make maximum use of this aid and depart from orthodox learning teaching approaches to enhance the thinking, social and personal skills of our children.

Dr. Mrs. Indira Lilamani Ginige
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(Curriculum Development)
Faculty of Science and Technology
National Institute of Education

2007.10.06

Contents

Descriptive Syllabus

Introduction

The abilities, skills, virtues and customs that should be developed in students, and which are based on the capabilities that those learning mathematics from grade 6 to grade 11 should aim at achieving, have been identified as a set of competencies and organized to suit the different grades. It is expected that students who study mathematics would approach all these competencies by the end of grade 11. To aid the students in approaching these competencies, competency levels and appropriate learning outcomes have been prepared for each competency. The competencies that students learning mathematics in grade 11 should approach, the appropriate competency levels, and the learning outcomes that are required to achieve the competency levels have been included in this manual. All these have been included in the manual such that they can be identified as the mathematics syllabus for grade 11. The subject content prepared to aid the students in achieving these learning outcomes, and the number of periods required to operate this subject content based learning teaching and assessment process have also been included in this syllabus. The new competency based syllabi related to mathematics have been in operation from 2007 in grade 6 and in 10, and a policy decision has been made to implement this syllabus in grade 11 from 2008.

The subject content of this syllabus has been organized so that the following aims of learning mathematics are achieved.

- Knowledge and Skills
- Communication
- Identifying Relationships
- Reasoning
- Problem Solving

It is expected that through this syllabus, mathematics will not just be limited to knowledge but that the skills required in practical life as well as virtues will be developed. In implementing the learning, teaching and exploring process of this competency based syllabus,

- the instances of meaningful discovery learning are increased and thus learning becomes more student centered
- guidance is provided for students to achieve the various competencies appropriate for their mental level and to develop these throughout their lives.
- the aims of learning, teaching and exploring become clearer.
- the goals of the teacher become more specific.
- since it is possible to identify how far each student has progressed towards achieving each competency level, it is easy for the teacher to provide the required feedback and feed-forward.
- the opportunity is given for students to develop the necessary mathematical concepts as well as the principles related to them.
- the teacher is able to step away from traditional teaching methods and enter into a transformation role.

When operating this mathematics syllabus in the classroom, under the sections that are considered topical, day to day phenomena should be used to create new teaching strategies.

It is more productive to implement this grade 11 mathematics syllabus by first resolving the problems that have been identified while operating the grade 6 and grade 10 syllabi in year 2007. Some of the problems that have been identified and remedial actions that could be taken have been included in this syllabus under the chapter 'school policies and programmes'.

The quality inputs that are required for the activities that have been proposed to provide students with experience within the learning, teaching and exploring process, have been included under each activity in the chapter named learning- teaching methodology.

In the learning, teaching and exploring process, since the opportunity to develop activities for each competency level is provided, assessment of the competency level that the student is approaching and evaluation of the student is facilitated. In addition, the following aspects

- the aims of learning mathematics
- the learning-teaching strategies
- the school policies and programmes
- proposed teaching sequence and number of periods
- the competency based syllabus

that aid in carrying out the various duties related to teaching mathematics in schools are also presented in this syllabus..

Aims of learning mathematics

The following objectives should be aimed at and achieved to further develop the mathematical concepts, creativity and sense of appreciation in students entering the senior secondary stage, so that their mathematical thinking, understanding and abilities are enhanced.

(1) The development of computational skills through the provision of mathematical concepts and principles as well as the knowledge of mathematical operations, and the development of the basic skills of solving mathematical problems with better understanding.

(2) The development of correct communication skills by enhancing the competencies of the proper use of oral, written, pictorial, graphical, concrete and algebraic methods.

(3) The development of relationships between important mathematical ideas and concepts, and the use of these in the study and improvement of other subjects. The use of mathematics as a discipline that is relevant to lead an uncomplicated and satisfying life.

(4) The enhancement of the skills of inductive and deductive reasoning to develop and evaluate mathematical conjectures and arguments.

(5) The development of the ability to use mathematical knowledge and techniques to formulate and solve problems both familiar and unfamiliar that arise in day to day life.

1. Knowledge and Skills

It is expected that students will acquire basic skills and gain understanding of concepts, principles and methods by learning mathematics. These can be used as tools to apply mathematical thinking in other fields, and also as a base for further mathematical activities. It is necessary to keep in mind the recent developments in technology when deciding on the skills and knowledge that the students should gain by memorizing or learning, as scientific calculators and symbolic processes which are affordable, more powerful and mechanical have an influence on senior secondary schools.

2. Communication

Due to the fact that mathematics has the power to communicate and represent ideas concisely and with clarity, the use of it in other disciplines has increased tremendously. Guaranteeing that students come to a common agreement regarding concepts and definitions should be an important component of the school curriculum.

This can be accomplished by providing students with the latitude to explain their ideas both orally and in writing, to think intuitively, and to defend their ideas. Through such activities, the skills of exchanging knowledge and working with amity in co-operation with others are guaranteed. These are considered as higher skills in the modern world of work.

3. Relationships

Students are often inclined to think of mathematics as a series of isolated and unrelated truths and processes. It is therefore important to educate them about the many relationships that exist even in the learning of mathematics through graphical, numerical, physical and algebraic representations or models. Students should recognize that mathematical thinking and modeling are used to solve problems in other fields such as the Biological, Physical and Social Sciences, Arts, Music and Commerce, as well as in day to day life. They should also understand the manner in which mathematics has been related to our culture both local and foreign, in the past as well as in the present.

4. Reasoning

The argument that has existed for years to pay greater attention to mathematics in the school curriculum is due to the belief that by learning mathematics, students will be able to develop clear and logical thought.

Although the principles of logic form the base for deductive reasoning in mathematics, there are many developments in mathematics based on induction, i.e., on conjectures that are proved by deductive reasoning, by first identifying patterns. The development of mathematics has occurred through the interactions that take place between various observations, identification of patterns, making assumptions and proving theorems. Students should be educated regarding these various aspects of mathematical thought and should also develop the skills related to each of them.

5. Problem Solving

For a student to become a productive citizen it is essential that he/she develops problem solving skills. Problem solving is a common investigative path through which students develop an awareness of the usefulness and power of mathematics. Students will face various issues when they attempt to use logical and creative thought to analyze a mathematical method by which a wrong conclusion has been arrived at. But in every situation, the students have the ability to solve problems that are within the scope of the knowledge they have gained. Suitable methods should be developed to evaluate and appreciate the success of these efforts by students and these evaluation methods should be incorporated into the regular evaluation scheme.

Grade 11 – Mathematics

Competencies, Competency Levels and Subject Content

Competencies and Competency Levels	Subject Content	Periods
Competency – 1 **Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.**		

1.1 Classifies the set of real numbers.	• Irrational Numbers
	  • Introduction (As non-terminating and non-recurring decimal numbers)
	• Set Notations
	  • Set of Natural Numbers
	  • Set of Integers
	  • Set of Rational Numbers
	  • Set of Irrational Numbers
	  • Set of Real Numbers
		04

1.2 Manipulates the basic mathematical operations related to surds.	• Simplification
	  • Surds
	  • Entire Surds
	• Rationalizing the Denominator
	(Only the form $\frac{a}{\sqrt{b}}$)
		04

Competency – 2 **Makes decisions for future requirements by investigating the various relationships in number patterns.**		

2.1 Investigates the various behavioral patterns of progressions using sequences of numbers.	• Geometric Progressions
	  • n^{th} Term
	  • Sum of n Terms
	  • Geometric Mean
		08

Competency - 5 **Makes successful transactions in the modern world of commerce by using percentages.**		

Competencies and Competency Levels	Subject Content	Periods
5.1 Uses percentages when making transactions in installments.	• Loan Installments • Hire Purchases (in installments and on the reducing balance)	03
5.2 Makes transactions by comparing schemes of interest.	• Compound Interest (Up to 3 stages)	03
5.3 Considers the share market to invest money.	• Joint Businesses • Shares	05
Competency – 6 **Easily solves mathematical problems in day to day life by using logarithms and calculators**.		
6.1 Simplifies expressions with indices.	• Integral Indices • Rational Indices	04
6.2 Simplifies expressions with logarithms.	• Logarithmic Laws (for powers and roots) • Expressions with Powers and Roots • Equations with Powers and Roots (without using logarithmic tables)	04

6.3 Simplifies expressions using logarithmic tables.	• Logarithms of Numbers less than 1
	• Expressions Including Numbers less than 1
	  • Multiplication
	  • Division (including powers and roots)
	  • Verifying Results using a Calculator
		04

Competency – 8 **Makes use of a limited space in an optimal manner by investigating the area.**		

Competencies and Competency Levels	Subject Content	Periods

8.1 Investigates the surface areas of various solids in the surrounding.	• Surface Area
	  • Cone
	  • Sphere
	  • Pyramid (Square Base)
		05

Competency – 10 **Gets the maximum out of space by working critically with respect to volume**.		

10.1 Explores the volumes of various solids.	• Volume
	  • Cone
	  • Sphere
	  • Pyramid (Square Base)
		05

Competency – 13 **Uses scale drawings in practical situations by exploring various methods**.		

13.1 Manipulates trigonometric relationships for daily requirements.	• Use of Trigonometric Ratios (by trigonometric tables)
	  • Sine
	  • Cosine
	  • Tangent
		12

Competency – 14 **Simplifies algebraic expressions by systematically exploring various methods.**		

14.1 Determines the cubes of binomial expressions.	• Cubes of Expressions where the Coefficient of the Unknowns is 1
	  • $(a+b)^3$
	  • $(a-b)^3$
	  • $(x\pm 2)^3$
		04

Competencies and Competency Levels	Subject Content	Periods
Competency - 16 **Solves problems encountered in day to day life by exploring the various methods of simplifying algebraic fractions.**		

16.1 Manipulates algebraic fractions under the basic mathematical operations.	• Expressions with Algebraic Fractions
	  • Addition
	  • Subtraction
	  • Multiplication
	  • Division
		04

Competency – 17 **Manipulates the methods of solving equations to fulfill the needs of day to day life.**		

17.1 Investigates how quadratic equations may be used to solve problems that arise in daily life.	• Solving Quadratic Equations
	  • Using Factors
	  • By Completing the Square
	  • Using the Formula
		07

Competency – 18 **Analyzes the relationships between various quantities related to real-life problems.**		
18.1 Solves problems involving the relationships between two quantities.	• Representation of the Solutions of Inequalities on the Number Line • Of the form $ax + b \gtreqless cx + d$ • Problems involving Inequalities	06
Competency – 20 **Easily communicates the mutual relationships that exist between two variables by exploring various methods.**		
20.1 Solves problems using graphical methods.	• Solutions of Pairs of Simultaneous Equations • Using Graphs	04

Competencies and Competency Levels	Subject Content	Periods

20.2 Analyses the features of a quadratic function by its graph.	• Graphs of Quadratic Functions
	  • Of the Form $y = ax^2 + bx + c$
	• Features of the Graph of a Quadratic Function
	  • Axis of Symmetry
	  • Behaviour of the Function
	  • Maximum/Minimum Point
	  • Roots (when $y = 0$)
	• Features of the Graph of a Function of the Form $y = (x \pm a)^2 + b$ (without drawing the graph)
		06

20.3 Communicates the mutual relationships that exist between two variables using matrices.	• Matrices
	  • Introducing Matrices (Up to 3×3)
	  • Addition of Matrices
	  • Subtraction of Matrices
	  • Multiplying a Matrix by an Integer
		06

Competency – 23 **Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures**.		

23.1 Determines the relationship between the areas of parallelograms and triangles that lie between the same parallel lines.	• Theorems (proof not required)
	  • Parallelograms on the same base and between the same parallel lines are equal in area
	  • If a parallelogram and a triangle stand on the same base and between the same parallel lines, the area of the triangle is half the area of the parallelogram
		06

Competencies and Competency Levels	Subject Content	Periods

23.2 Uses the relationship between the areas of triangles with common bases and common vertices to make decisions.	• Theorems (proof not required)
	  • Triangles on the same base and between the same parallel lines are equal in area
	  • The areas of triangles of equal altitude are proportional to one another as their bases
		06

23.3 Investigates the relationship between the sides of a triangle and parallelism.	• The theorem 'A straight line drawn parallel to one side of a triangle cuts the other two sides, or those sides produced, proportionally' and its converse (proof not required)	06

23.4 Investigates the equi-angularity of triangles.	• Equi-angular Triangles
	  • Ways in which triangles are equi-angular
	  • Application of the theorem 'If two triangles are equiangular to one another, their corresponding sides are proportional' and its converse (proof not required)
		06

23.5 Examines the relationship between the sides of a right-angled triangle.	• Pythagoras' Theorem (proof not required) • Application and Problems	06
23.6 Examines the results when the sides of a triangle are divided proportionally.	• Mid-point Theorem (proof not required) • Theorem and its Converse	06
Competency – 24 **Thinks logically to make decisions based on geometrical concepts related to circles.**		

Competencies and Competency Levels	Subject Content	Periods

24.1 Investigates the quadrilaterals that can be inscribed in a circle.	• Cyclic Quadrilaterals
	  • The Theorem 'The opposite angles of a cyclic quadrilateral are supplementary' (proof and application)
	  • Application of the converse of the above theorem
		03

24.2 Examines the relationship between the exterior and interior angles of a cyclic quadrilateral.	• Application of the theorem 'If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral' (proof not required)	03

24.3 Formally confirms the behaviour of angles related to the tangents of a circle.	• Tangents to a Circle
	  • Application of the theorem 'The straight line drawn through a point on a circle and perpendicular to the radius to the point of contact is a tangent to the circle' (proof not required)
	  • Application of the converse of the above theorem
		03

24.4 Examines the properties of tangents drawn from an exterior point to a circle.	• The proof and application of the theorem 'If two tangents are drawn to a circle from an exterior point then
	  • the tangents are equal to each other in length
	  • the tangents subtend equal angles at the centre
	  • the angle between the tangents is bisected by the straight line joining the exterior point to the centre'
		03

Competencies and Competency Levels	Subject Content	Periods
24.5 Examines the relationship between the angle that a tangent to a circle makes with a cord drawn from the point of contact, and the angles in the alternate segments of the circle.	• Introduction of the Alternate Segment of a Circle • Application of the theorem 'The angles which a tangent to a circle makes with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle' (proof not required)	03
Competency – 27 **Analyzes according to geometric laws, the nature of the locations in the surroundings.**		

27.1 Constructs tangents to a circle by using the relationships between the angles related to tangents.	• Construction of Tangents
	  • At a point on the circle
	  • From an exterior point
		03

27.2 Investigates the methods of partitioning a straight line segment.	• Division of a Line Segment into Equal Parts	03
Competency – 28 **Facilitates daily work by investigating the various methods of representing data.**		

28.1 Represents data graphically.	• Histogram (with unequal class intervals)
	  • Discrete Data
	  • Continuous Data
		02

28.2 Develops relationships between the various graphs that represent data.	• Frequency Polygon	03

28.3 Represents the relationships between the cumulative frequency and the boundaries of a group of data.	• Cumulative Frequency Curve (for grouped and ungrouped data)
	  • Quartiles
	  • Inter-quartile Range
		03

Competencies and Competency Levels	Subject Content	Periods
Competency – 30 **Manipulates the principles related to sets to facilitate daily activities.**		

30.1 Identifies the regions of the union and the intersection of sets using Venn diagrams.	• Venn diagrams (up to 3 sets)
	  • Using Set Operations
	  • Identifying Regions
	  • Solving Problems
		06

Competency – 31 **Analyzes the likelihood of an event occurring to predict future events.**		

31.1 Interprets daily occurrences using the various methods of representing the likelihood of an event occurring.	• Sample Space
	  • Using Tree Diagrams (up to 2 stages)
	  • Solving Problems
		07

Relationship between Subject Themes and Content

Content	Learning Outcomes
1.0Numbers	
1.1 Set of Real Numbers • Classification of Numbers	• Identifies non-terminating and non-recurring decimals as irrational numbers
• Representation using Set Notations	• Represents the set of natural numbers, set of integers, set of rational numbers, set of irrational numbers and the set of real numbers using set notations
1.2 Surds • Simplification	• Identifies surds as irrational numbers • Simplifies entire surds into surds
• Rationalizing the Denominator	• Simplifies surds by rationalizing the denominator (only of the form $\frac{a}{\sqrt{b}}$)
1.3 Geometric Progression • Introduction • n^{th} term • Sum of n terms	• Identifies geometric progressions • Finds the n^{th} term of a geometric progression • Finds the sum of n terms of a geometric progression • Finds the number of terms when the sum is given
• Geometric Mean	• Finds the geometric mean of the terms of a geometric progression
1.4 Percentages • Loan Installments and Hire Purchases	• Finds the installments by calculating the interest for the reducing balance • Finds the interest rate when the installment on the reducing balance is given
• Compound Interest	• Calculates the compound interest up to three stages
• Joint Businesses and Shares	• Solves problems related to shares and joint businesses
1.5 Logarithms • Indices	• Simplifies expressions with rational indices

Content	Learning Outcomes
• Logarithmic Laws	• Uses the logarithmic laws relevant to powers and roots • Simplifies expressions involving powers and roots without using the logarithmic tables • Solves equations involving powers and roots without using the logarithmic tables
• Logarithmic Tables	• Simplifies expressions involving multiplication, division, powers and roots using logarithmic tables (including numbers less than 1) • Confirms results with a calculator
2.0 Measurements	
2.1 Area • Surface Area	• Finds the surface area of a cone • Finds the surface area of a sphere • Finds the surface area of a right pyramid with a square base (using the formulae)
2.2 Volume • Volume of Solids	• Finds the volume of a cone • Finds the volume of a sphere • Finds the volume of a right pyramid with a square base (using the formulae)
2.3 Scale Drawings • Trigonometric Tables	• Uses Trigonometric Tables to find heights and distances • Uses Trigonometric Tables to find the value of an angle (angles of elevation, angles of depression and bearings included. Only containing one variable and in one plane)
3.0 Algebra	
3.1 Binomial Expressions • Cubes	• Finds the cubes of binomial expressions where the coefficients of the unknowns are 1 Such as $(x \pm 2)^3$ and $(a \pm b)^3$

Content	Learning Outcomes
3.2 Algebraic Fractions	
• Simplifying	• Simplifies expressions with algebraic fractions (including multiplication and division)
3.3 Quadratic Equations	
• Solving	• Solves quadratic equations by various methods (by factorizing, by completing the square and by using the formula)
3.4 Inequalities	
• Solving	• Solves inequalities with one variable of the form $ax + b \leq cx + d$, $ax + b \geq cx + d$
• Representation of Solutions	• Represents the solutions of inequalities of the above form on a number line
• Solving Problems	• Solves problems that include inequalities
3.5 Graphs	
• Solutions of a Pair of Simultaneous Equations	• Finds the solutions of a pair of simultaneous equations using graphs
• Quadratic Functions	• Sketches the graph of functions of the form $y = ax^2 + bx + c$ for a given domain • Determines the features of graphs of the above form (behaviour of the graph, maximum/minimum value and the axis of symmetry) • Determines the features of graphs of functions of the form $y = (x \pm a)^2 + b$ without sketching the graph
• Finding Roots	• Finds the roots of $y = 0$
3.6 Matrices	
• Introduction	• Identifies matrices up to order 3×3
• Addition	• Adds two matrices
• Subtraction	• Subtracts two matrices
• Multiplication	• Multiplies a matrix by an integer

Content	Learning Outcomes
4.0 Geometry	
4.1 Relationships between the areas of triangles and parallelograms lying between the same two parallel lines	• Applies the theorem 'Parallelograms on the same base and between the same parallel lines are equal in area' (theorem without proof) • Applies the theorem 'If a parallelogram and a triangle stand on the same base and between the same parallel lines, the area of the triangle is half the area of the parallelogram' (theorem without proof) • Applies the theorem 'Triangles on the same base and between the same parallel lines are equal in area' (theorem without proof) • Applies the fact that areas of triangles of equal altitude are proportional to one another as their bases, to solve problems
4.2 Equi-angular triangles and similar triangles	• Identifies the instances when two triangles are equi-angular • Applies the theorem 'If two triangles are equi-angular to one another, their corresponding sides are proportional', and its converse (theorem without proof) • Applies the theorem 'A straight line drawn parallel to one side of a triangle cuts the other two sides, or those sides produced, proportionally', and its converse (theorem without proof)
4.3 Right-angled Triangles • Pythagoras' Theorem	• Solves problems by applying Pythagoras' theorem (theorem without proof)
4.4 Mid-point Theorem	• Applies the mid-point theorem and its converse (theorem without proof)
4.5 Cyclic Quadrilaterals	• Proves and applies the theorem 'The opposite angles of a cyclic quadrilateral are supplementary' • Applies the converse of the above theorem without proof

Content	Learning Outcomes
	• Applies the theorem 'If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral' (theorem without proof)
4.6 Tangents	• Applies the theorem 'The straight line drawn through a point on a circle and perpendicular to the radius to the point of contact is a tangent to the circle', and its converse (theorem without proof)
4.7 Tangents drawn from an exterior point	• Proves and applies the theorem 'If two tangents are drawn to a circle from an exterior point then (i) the tangents are equal to each other in length. (ii) the tangents subtend equal angles at the centre (iii) the angle between the tangents is bisected by the straight line joining the exterior point to the centre'
4.8 Tangents and Angles	• Applies the theorem 'The angles which a tangent to a circle makes with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle' (theorem without proof)
4.9 Constructions	• Constructs a tangent at a point on the circle • Constructs tangents from an exterior point to a circle
	• Divides a line segment into equal parts
5.0 Statistics	
5.1 Representation of Grouped Data	• Draws a histogram for discrete and continuous data in unequal class intervals
	• Draws the frequency polygon for a given set of data. Draws the cumulative frequency curve for a given set of data

Content	Learning Outcomes
5.2 Interpretation of Data	• Determines the quartiles and the inter-quartile range for data that is represented by a cumulative frequency curve (For grouped and ungrouped data)
6.0 Sets and Probability	
6.1 Sets	• Identifies the regions of the union and the intersection of sets using Venn diagrams (up to 3 sets) • Solves problems using Venn diagrams (up to 3 sets)
6.2 Probability	• Represents the sample space by a tree diagram (not more than 2 stages) • Solves problems using tree diagrams

Relationship between the Teaching Sequence and the Competency Levels

Content	Competency Levels	Number of Periods
Term 1		
01 Real Numbers	1.1, 1.2	08
02 Indices and Logarithms I	6.1, 6.2	05
03 Indices and Logarithms II	6.3	07
04 Surface Area of Solids	8.1	05
05 Volume of Solids	10.1	05
06 Binomial Expressions	14.1	04
07 Algebraic Fractions	16.1	04
08 Area of Rectilinear Plane Figures between Parallel Lines	23.1, 23.2	12 (**50)**
Term 2		
09 Percentages	5.1, 5.2	06
10 Business and Shares	5.3	05
11 Mid-point Theorem	23.6	06
12 Equi-angular Triangles	23.3, 23.4	12
13 Quadratic Equations	17.1	07
14 Graphs	20.1, 20.2	10
15 Data Representation and Interpretation	28.1, 28.2, 28.3 2.1	06 08 (**60)**
16 Geometric Progression		
Term 3		
17 Pythagoras' Theorem	23.5	06
18 Trigonometry	13.1	12
19 Matrices	20.3	06
20 Inequalities	18.1	06
21 Cyclic Quadrilaterals	24.1, 24.2	10
22 Tangents	24.3, 24.4, 24.5	10
23 Constructions	27.1, 27.2	06
24 Sets	30.1	06
25 Probability	31.1	07 **(69)**
		Total 179

School Policies and Programmes

The mathematics syllabus has been prepared not only with the objective of inculcating knowledge and skills but also to highlight the deeper aims of communication, relationships, logical argument and problem solving. The latterly mentioned four aims aid more effectively in the development of behavioral and thinking skills. Mathematics is a subject that should not be restricted to just the syllabus or the classroom. It should be made into an active force within the school culture itself as it is a language; a science; an art; a tool to be used in thought, in calculations and in creations.

It is important therefore to organize school programmes so that the cultural values embedded in mathematics are developed in students. The following co-curricular programmes will aid in this.

1. Wall newspapers
2. Mathematics Laboratory
3. Mathematics Library
4. Exhibitions
5. Mathematics Societies
6. Quizzes
7. Mathematics Magazines
8. Mathematics Days
9. Mathematics Camps
10. Activity Cells
11. Excursions

In operating this co-curricular programme, the school management should aim at obtaining the help of the community when required and also getting experts to teach some of the subject content.

If your school does not have a trained mathematics teacher for this grade, it is appropriate to engage a teacher who has the required subject knowledge and a penchant for teaching mathematics for this task. Mathematics teacher should continuously update their knowledge on subject material and teaching methodologies. For this, it is appropriate to get advice from the zonal mathematics director/ mathematics in-service advisors and senior graduate teachers. It is also necessary to participate in training sessions.

It is hoped that the school management will work towards allocating one of the free periods in the time table under the new reforms, for mathematics.

It is most likely that it will not be possible to complete a proposed exploration activity within a period due to most activities being longer than 40 minutes. In such instances, the next mathematics period should be used to complete the activity.

To facilitate the tasks of the students as well as the teacher, chairs should be placed in a semi-circle during exploration activities. The quality inputs required for the activities are given under each activity in the chapter on learning teaching methodology. It would be easier to buy all the quality inputs for the year at once at the beginning of the year.

In guiding the learning teaching process, it is important to provide instructions by paying special attention to the following:

- Determining whether the students are engaged in exploration according to the E-5 model
- Carrying out assessment and evaluation at the appropriate times

It is also expected that attention will be paid to the following, which are proposed as remedies to the problems encountered when the competency based syllabi were operated in the school system in year 2007 in grades 6 and 10.

- Groups should be organized according to the number of students in the class. (If tasks have been assigned for 4 groups, the number of groups should be taken as a multiple of this number, depending on how many more/less students are in the class).

- The tasks in the instruction leaflet on exploration should be distributed randomly among the groups.

- Instead of appointing a leader, the opportunity should be provided for a person within the group itself to volunteer to be the leader.

- Students should first be engaged in the exploration process, and once the relevant mathematical concepts have been grasped through this process, the remaining time should be used for exercises.

- When a review is being done after the exploration process is concluded, students should be provided with the opportunity to write a summarized note on the lesson in their exercise books.

- Depending on the number of evaluations that should be carried out in a term, apart from the instruments for the extension of the learning – teaching process, to carry out the evaluations, appropriate activities should be selected and marks should be allocated.

Learning – Teaching Methodology

Introduction

In deciding upon the learning teaching methodology relevant to the course, attention has been paid to the planning of learning-teaching activities that facilitate building up of student competencies based on exploration. In preparing for a competency-based education of this manner, an obvious change in the role of the teacher is expected.

The transmission role practiced in our classroom from way back and the more recently introduced transaction role is evident in the classroom even today. When taking into consideration the deterioration of the thinking, social and personal skills of school leavers, no effort is needed to understand that there is a need for the development of the learning-teaching methodology and how it should be effected.

In the transmission role, the teacher is considered an individual who knows everything that the student should learn, and his task is that of transmitting knowledge to the student who is considered as one who does not know anything. This learning-teaching process that takes the guise of lectures is restricted only to the flow of knowledge from the teacher to the student, and does not make an adequate contribution either to the stimulation of student thinking or to the development of his personal and social skills.

The dialogue initiated by the teacher within the class is the initial stage of the transaction role. Apart from the ideas that flow from the teacher to the class and from the class to the teacher, as a result of the student-student interactions that takes place subsequently, these dialogues transform gradually into discussions. The teacher is continuously involved in the task of questioning in order to take the student from the known to the unknown, from the simple to the complex and from the concrete to the abstract.

While, in competency-based education, student tasks occupy a powerful position, the teacher takes on the role of a resource person who mediates in order to provide every student in the class with a competency that is at least proximate to the required competency. For this, the basic functions that the teacher is expected to perform include initiating the activity in a manner that will kindle interest, planning the learning environment with the necessary materials and other facilities, closely observing how students learn, identifying student abilities and inabilities and promoting student learning through the provision of feedback and feed-forward, paying close attention to student presentations and discussions, as well as preparing instruments for the extension of learning beyond the classroom. The teacher's role based essentially upon the tasks mentioned above is called the transformation role.

The series of activities that can be used in the implementation of the descriptive curriculum introduced in the first part of this teacher's instructional manual, has been included in its second part. Each of these activities has been developed so as to contain a minimum of three steps. It is expected to get the student involved in the learning process through the first step of the activities. Therefore, this step is termed the "engagement" step. As an introduction to this step, the teacher assumes the transaction role and engages in a

dialogue with the students. Subsequently, along with the transformation of this dialogue to a discussion, the students engage in exploration and are provided the opportunity to recall the pre-knowledge related to the basic competency they should develop, and to acquire a hint regarding the future of the activity. The teacher possesses a host of strategies that can be used in these exchanges of ideas. Some of the devices at the disposal of the teacher for the exchange of these ideas are questions/stimulants like pictures, newspaper advertisements and flash cards/puzzles or case studies/dialogues, role play, poems, songs and demonstrations, video tapes or audio tapes. In summary, the first step of the activities is implemented with the aim of actualizing the following three objectives.

- Winning over of the attention of the class.
- Providing the students with the opportunity for students' recall of the necessary pre-knowledge.
- Introducing the elements of the exploration the students are expected to be directed to under the second step of the activity.

It is with the objective of providing the students with the opportunity for exploration that the second step of the activity has been planned. Students base their exploration on a special leaflet prepared for the purpose. The teacher has to plan this step to enable the students to engage in co-operative learning in groups, through the exploration of various aspects of the problem. Two of the most important features of this step are the use of the resource materials provided and conscious involvement in group discussions. As a result of involvement in group activities throughout a long period of time, students will develop a number of essential skills like self-discipline, listening to others, working co-operatively with others, helping others, time management, producing high quality creations, honesty etc.

In directing students towards exploration, the teacher should avoid making decisions regarding leadership in the group, but he/she should prepare the background necessary for a leader to emerge. Accordingly, when opportune, the students will have the privilege of taking on leadership, based on inherent abilities.

During the 3rd step of the activity, every group will have the opportunity of presenting the results of its exploration for the enlightenment of the others. What the teacher has to do here is to encourage student involvement in group presentations. It would be effective if students are directed, so as to ensure that every member is given responsibility in the planning of the presentation. An important aspect of this step, related to the explanation of student findings, is the creation of the opportunity for the voice of students to be heard in the classroom, where commonly the voice of the teacher dominated.

After the explanation of the findings in the third step of the activities, students should be directed to elaboration. Each group is given the opportunity to provide constructive suggestions on its findings first, and subsequently, members of other groups are given this opportunity. Anyway, the final review is the responsibility of the teacher. During this review, the teacher is expected to clarify all the important points relevant to the students' exploration as well as to instill in students the right understanding of the concepts and rules.

The main responsibility of the teacher in this teaching methodology is to monitor continuously whether the classroom learning-teaching process is being implemented successfully and as expected. While assessment and evaluation should be made use of for this purpose, the teacher is provided the opportunity to carry this out, through planned activities, in the learning teaching process itself. The teacher is given the opportunity for assessment while the students are involved in exploration during the second stage of the activity and for evaluation when the students are involved in explanation and elaboration during the third stage. A detailed inquiry into assessment and evaluation will be provided later on in this document.

The teacher is provided direction on the transformation role by the learning – teaching methodology described so far. While priority is given to group exploration here, the teacher is also afforded the opportunity for transaction, discussion and short lectures. While there is room for transaction and discussion in the initial stage, the teacher may also give a short lecture to confirm the concepts, under review, in the final stage. In the development of the learning-teaching methodology related to this curriculum, the first to be introduced under the curriculum reforms for the new millennium, the attention paid to the important aspects of the transmission and the transaction roles of the teacher, apart from the transformation role, is a special feature of this methodology.

Activity Continuum

01.Real Numbers I

Competency 1 : Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.

Competency Level 1.1 : Classifies the set of real numbers.

Activity 1.1 : Let us separate numbers into sets according to their common characteristics; Let us name them.

Time : 60 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 1.1.1.
- Demy papers and pastels.
- Calculators.

Learning –Teaching Process:

Step 1.1.1 :
- Ask the students to mention various numbers and write them on the blackboard.
- Inquire from the students about the sets that the numbers belong to.
- Lead a discussion and highlight the following facts.

  - That numbers can be separated into sets according to their common features
  - That these sets are defined as natural numbers, integers and rational numbers
  - That rational numbers are of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$
  - That a rational number can be expressed as a terminating decimal or a recurring decimal

(10 minutes)

Step 1.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 1.1.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That numbers such as $\sqrt{3}, \sqrt{5}$ cannot be written as terminating decimals or recurring decimals and that these numbers are defined as irrational numbers
- That the set of natural numbers is denoted by N, the set of integers by Z and the set of rational numbers by Q
- That the numbers which are not rational numbers are irrational numbers and that the set of irrational numbers is denoted by Q'
- That the set to which the natural numbers, integers, rational numbers and irrational numbers all belong is the set of real numbers
- That all these numbers may be represented on the number line
- That the set of real numbers is denoted by R

(30 minutes)

Criteria for Assessment and Evaluation:

- Names the various sets of numbers which are subsets of the set of real numbers.
- Accepts that there are decimal numbers in the set of real numbers which are neither terminating decimals nor recurring decimals.
- Classifies numbers that are given as rational numbers or irrational numbers.
- Participates actively in the plenary discussions.
- Identifies the parts by analyzing the whole.

Annex 1.1.1

Instructions for group exploration

Let us separate numbers into sets according to their common characteristics; Let us name them

A

3 $\sqrt{3}$ -2 $\sqrt{2}$ 0 -4 5 $\sqrt{11}$ -1 12 $\sqrt{7}$ 27

B

$\frac{1}{2}$ $\sqrt{5}$ $\frac{3}{5}$ $\sqrt{2}$ $\frac{3}{4}$ $\sqrt{3}$ $\frac{2}{7}$ $\sqrt{7}$ $\frac{3}{19}$ $\frac{3}{20}$ $\sqrt{23}$ $\sqrt{11}$ $\frac{7}{10}$

C

0.5 $\sqrt{13}$ 0.75 0.2 $\sqrt{11}$ $\sqrt{17}$ $\sqrt{5}$ $\sqrt{7}$ 0.3 0.13 0.6 $\sqrt{2}$ 0.125

- Focus your attention on the card containing numbers which your group receives.
- Select and separate out the numbers that do not belong to the sets of numbers that you have used before.
- Find the values of the numbers that have been separated out using a calculator.
- Write down more numbers of this type, find their values using a calculator and examine whether the values that you obtain are terminating or recurring decimals.
- Discuss how these numbers differ from the numbers that you are familiar with.
- Name the sets to which all the numbers you know and use belong and propose suitable symbols for them.
- Prepare for a creative presentation at the plenary session.

01. Real Numbers II

Competency 1 : Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.

Competency Level 1.2 : Manipulates the basic mathematical operations related to surds.

Activity 1.2 : Let us simplify expressions with surds.

Time : 120 minutes.

Quality Inputs :
- An enlarged copy of the chart in Annex 1.2.1.
- Three copies of the instruction leaflet on exploration included in Annex 1.2.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 1.2.1 :
- Present the chart to the class and make inquires from the students about rational numbers and irrational numbers.
- Lead a discussion and highlight the following facts.

• That the square roots of whole numbers that are perfect squares can be determined by their prime factors • That the square roots of numbers that are not perfect squares may be represented using the symbol $\sqrt{\ }$ • That such numbers are irrational numbers and are defined as surds

(15 minutes)

Step 1.2.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(25 minutes)

Step 1.2.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That surds which are identical square roots can be added and subtracted
  - That when two surds are multiplied or divided, the answer can be written as a single surd
  - That an entire surd is the square root of a number which can be separated into two factors, one of which is a perfect square
  - That when a surd is multiplied by a whole number, it can be written as an entire surd

(30 minutes)

Step 1.2.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(20 minutes)

Step 1.2.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That simplification of a fraction that has a denominator which is an irrational number is difficult because division has to be performed with a large decimal number
  - That the denominator can be transformed into a rational number by multiplying the numerator and the denominator by the surd in the denominator
  - That because the surd in the fraction that is obtained finally appears in the numerator, simplification can be easily done by substituting the given value for the surd and then multiplying

(30 minutes)

Criteria for Assessment and Evaluation:

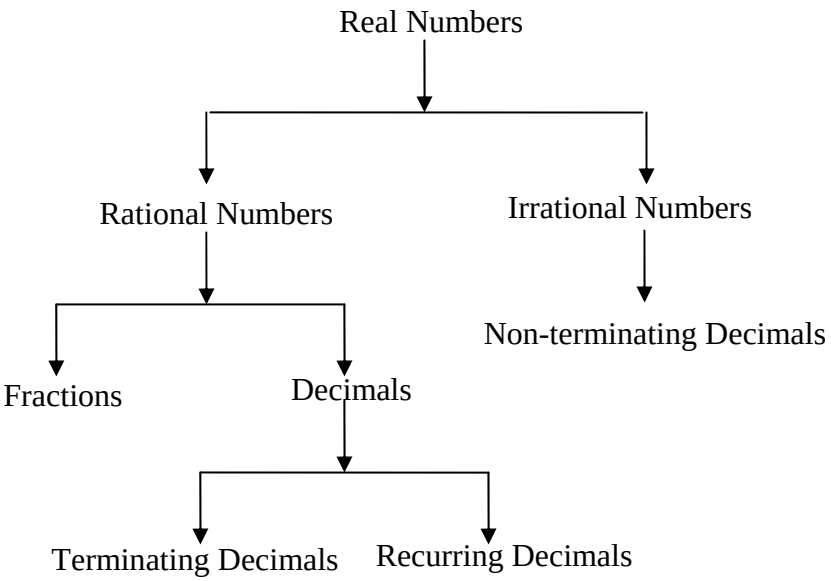
- Describes the steps that need to be performed to rationalize the denominator of a fraction which has a surd as the denominator.
- Accepts that simplification is made easier by rationalizing the denominator of fractions with surds as denominators.
- Simplifies expressions with surds.
- Gains understanding through observation.
- Uses appropriate methods to simplify complex situations.

Annex 1.2.1

Chart

$\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{25} = 5$ } positive whole numbers

$\sqrt{2} = 1.4142....$, $\sqrt{3} = 1.7321....$, $\sqrt{5} = 2.2361....$ } non - terminating decimals (surds) (Irrational Numbers)



Annex 1.2.2

Instructions for group exploration

Let us simplify expressions with surds

Part 1

- Carefully study the following simplifications.

- $3\sqrt{x} + 4\sqrt{x} = 7\sqrt{x}$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

- Focus your attention on the mathematical operation obtained by your group from the following mathematical operations.
  - Addition
  - Multiplication
  - Division
- Write down any two surds that you like and simplify them under the relevant mathematical operation.
- By simplifying, do you obtain the solution as a single surd?
- If not, discuss the reason why, and write down two surds that can be simplified under the given operation and simplify them.
- Considering the method used to obtain the answer, prepare to make a presentation on what has been discovered regarding the simplification of surds.
- Factor the number 20 in $\sqrt{20}$ such that one factor is a perfect square.
- Find the square root of the perfect square and write $\sqrt{20}$ in a different form.
- Develop a relationship between the obtained solution and the surd and prepare to describe the method of obtaining the relationship.

Part II

(A) The value of $\frac{8}{\sqrt{2}}$ if $\sqrt{2} = 1.4142$

(B) The value of $\frac{7}{\sqrt{3}}$ if $\sqrt{3} = 1.7321$

(C) The value of $\frac{4}{\sqrt{5}}$ if $\sqrt{5} = 2.2361$

- Focus your attention on the exercise received by your group from the above exercises.
- Substitute the given value for the surd in the denominator and see whether the fraction can be simplified.
- If it is difficult, propose a method to remove the surd from the denominator using mathematical operations, so that the simplification is made easier.
- Now find the solution to the exercise easily by substituting the given value for the surd in the expression that you obtained.
- Prepare to explain to the others an easy method to simplify a number that has an irrational number as the denominator, which involves re-writing the number in a suitable manner.

02. Indices and Logarithms I - I

Competency 6 : Easily solves mathematical problems in day to day life by using logarithms and calculators.

Competency Level 6.1 : Simplifies expressions with indices.

Activity 6.1 : Let us simplify expressions with indices.

Time : 120 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 6.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 6.1.1 :
- Pose several problems on multiplying and dividing powers that have the same base as well as problems on writing negative indices as positive indices, to the class.
- Use these problems to inquire from the students about simplifying the indices of expressions with indices.
- Lead a discussion and highlight the following facts.

• That when multiplying powers that have the same base, the indices should be added • That when dividing powers that have the same base, the difference of the indices should be obtained • That when a negative indexed power is in the numerator (or denominator), it can be changed into a positive indexed power by taking it to the denominator (or numerator)

(15 minutes)

Step 6.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 6.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That when simplifying expressions with indices, powers with integral indices as well as rational indices may be simplified into a single indexed power
  - That when simplifying expressions involving powers of powers, the solution can be obtained by multiplying the relevant indices
  - That $\left(a^m\right)^n = a^{mn}$

(25 minutes)

Step 6.1.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for presentations at the plenary session.

(30 minutes)

Step 6.1.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That simplifying powers with rational indices can be done easily by writing the base as an appropriate power
  - That a power with a rational index can be simplified easily only if the base can be written as an appropriate power
  - That when simplifying a power which has a fraction as its base, the solution can be given by simplifying the numerator and the denominator separately
  - That when the index is a mixed number, simplification is made easier by writing it as an improper faction

(20 minutes)

Criteria for Assessment and Evaluation:

- Describes how indices are manipulated when powers are being simplified.
- Accepts that the base and the index of a number have to be applied correctly when carrying out a simplification.
- Simplifies expressions with a power of a power.
- Considers the correct responses of other groups.
- Identifies easy methods to do calculations.

Annex 6.1.1

Instructions for group exploration

Let us simplify expressions with indices

Part I

- Carefully study the following problems.
  (i) $(2^2)^3$ (ii) $(a^m)^2$
- Expand and write these expressions.
- Apply your knowledge on indices and write each expanded expression as a single indexed power.
- Accordingly, write the expression $(a^m)^n$ as a single indexed power with base a .

(i) $(a^m)^n$
(ii) $(a^{-m})^n$
(iii) $(a^m)^{-n}$

- Focus your attention on the power that your group received from the above powers.
- Substitute whole numbers that are less than 5 for *m* and *n* in the power that your group received and simplify the power as much as possible.
- Substitute a whole number less than 5 for *m* and either $\frac{1}{2}, \frac{2}{3}$ or $\frac{3}{4}$ for *n* and simplify the power as much as possible.
- If you cannot simplify the power with the fraction you substituted for *n*, substitute a different fraction for *n* and simplify the expression.
- Using this, get ready to present at the plenary session, what is necessary to simplify a power which is raised to a fractional power.

Part II

(i) $a^{\frac{1}{2}}$ and $b^{-1\frac{1}{2}}$ (ii) $a^{\frac{2}{3}}$ and $b^{-\frac{4}{3}}$ (iii) $a^{\frac{3}{4}}$ and $b^{-\frac{1}{3}}$

- Focus your attention on the pair of powers that your group receives from the above pairs of powers.

- Substitute numbers that can be represented as powers for a and b and simplify the obtained powers as much as possible.
- If the obtained powers could not be simplified, discuss the reasons for it.
- Substitute values for a and b until an instant that can be simplified is obtained and simplify the powers.
- Use the method you discovered to simplify $\left(\frac{27}{64}\right)^{\frac{2}{3}}$.
- Prepare to creatively present the findings of your group at the plenary sessions.

02. Indices and Logarithms I – II

Competency 6 : Easily solves mathematical problems in day to day life by using logarithms and calculators.

Competency Level 6.2 : Simplifies expressions with logarithms.

Activity 6.2 : Let us simplify expressions using the logarithmic laws.

Time : 100 minutes.

Quality Inputs :
- An enlarged copy of the chart included in Annex 6.2.1.
- Four copies of the instruction leaflet on exploration included in Annex 6.2.2.
- Demy papers and pastels.

Learning – Teaching Process :

Step 6.2.1 :
- Present the chart of expressions with logarithms to the class.
- Lead a discussion on the relationship between indices and logarithms and highlight the following facts.

- That $m = a^r \Leftrightarrow \log_a m = r$
- That $\log_a a = 1$
- That $\log_a (mn) = \log_a m + \log_a n$
- That $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(10 minutes)

Step 6.2.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (15 minutes)

Step 6.2.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.

- Engage in a review so that the following facts are highlighted.

• That the logarithm of a number in the form of a power can be obtained by using the logarithmic law related to multiplication • That $\log_a m^r = r \log_a m$ • That $\log_a \sqrt{m} = \frac{1}{2}\log_a m$

(25 minutes)

Step 6.2.4 **:**
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(25 minutes)

Step 6.2.5 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the logarithmic laws can be used to simplify expressions or solve equations that involve logarithms • That when using the logarithmic laws to simplify expressions, all the terms should be in the same base • That when the left hand side and right hand side of an equation are logarithms with the same base, an equation without logarithms can be developed

(25 minutes)

Criteria for Assessment and Evaluation:
- Describes the method of finding the logarithm of a power or a square root.
- Accepts that the logarithmic laws can by used to express complex expressions of logarithms in simpler forms.
- Simplifies expressions with logarithms using the logarithmic laws.
- Finds solutions that are based on theory to complex problems.
- Engages in learning by applying knowledge that is newly gained.

Annex 6.2.1

Chart of Expressions with Logarithms

- $32 = 2^5 \Rightarrow \log_{\square} \square = \square$
- $\log_a a = \square$
- $\log_a (mn) =$ _ _ _ _ _ _ + _ _ _ _ _ _
- $\log_a \left(\frac{m}{n}\right) =$ _ _ _ _ _ _ _
- $\log_{10} 25 + \log_{10} 4 = \log_{10}$ _ _ _ _ _ _ = _ _ _ _ _ _ _

Annex 6.2.2

Instructions for group exploration

Let us simplify expressions using the logarithmic laws

Part I

- Study the following simplification.

$$\begin{aligned}\log_a 3^4 &= \log_a (3\times 3\times 3\times 3)\\ &= \log_a 3 + \log_a 3 + \log_a 3 + \log_a 3\\ &= 4\log_a 3\end{aligned}$$

- Focus your attention on the expressions received by your group from the following expressions.

	A	B	C	D
1	$\log_2 2^5$	$\log_4 4^3$	$\log_5 5^2$	$\log_3 3^4$
2	$\log_a m^3$	$\log_a x^2$	$\log_a n^4$	$\log_a t^3$

- Considering the simplification that you studied above find the value of the first expression.
- Use the understanding you gain to simplify the second expression.
- Find an equivalent expression for $\log_a m^r$ by focusing your attention on the result obtained above.
- Develop an equivalent expression for $\log_a \sqrt{m}$ as well.
- Prepare to explain to others how you obtained the above expressions.

Part II

- Study the following simplification.

$$\frac{1}{2}\log_a 25 = \log_a 25^{\frac{1}{2}} = \log_a 5$$

- Focus your attention on the section that your group received from the expressions and equations given below.

A	$\frac{1}{2}\log_6 81 + 2\log_6 4 - \log_6 4$	$\log 2 + 3\log 2 = \log 2^x$
B	$2\log_{10} 5 + \log_{10} 8 - \frac{1}{3}\log_{10} 8$	$\log 3^x - \log 3^2 = 4\log 3$
C	$\frac{1}{2}\log_4 64 + 2\log_4 2 - \log_4 2$	$\frac{1}{2}\log 25 + \log x = \log 10$
D	$5\log_8 2 + \frac{1}{2}\log_8 100 - \log_8 5$	$2\log x + \log 3 = \log 12$

- Simplify the first expression by considering the simplification that you studied above.
- Discuss how the logarithmic laws can be applied to solve the given equation and solve it.
- Prepare for a presentation at the plenary session.

03. Indices and Logarithms II

Competency 6 : Easily solves mathematical problems in day to day life by using logarithms and calculators.

Competency Level 6.3 : Simplifies expressions using the logarithmic tables.

Activity 6.3 : Let us facilitate simplification by using logarithmic tables.

Time : 90 minutes.

Quality Inputs :
- A Logarithmic Table.
- Four copies of the instruction leaflet on exploration included in Annex 6.3.1.
- Calculators.
- Demy papers and pastels.

Learning – Teaching Process:

Step 6.3.1 :
- Show a logarithmic table to the students and inquire from them how the logarithm of a number is read and how the product of two numbers is found using logarithms.
- Lead a discussion and highlight the following facts.

- That the characteristic of the logarithm can easily be obtained using the scientific notation of the number
- That only the logarithms of numbers between 1 and 10 can be obtained using the logarithmic tables
- That the logarithmic value of a number which is given in the logarithmic tables is the index obtained when the number is written as a power of 10
- That when $P = 10^x$, $\lg P = x$
- That $\lg(AB) = \lg A + \lg B$
- That $\lg\left(\frac{A}{B}\right) = \lg A - \lg B$
- That $\sqrt{a} = a^{\frac{1}{2}}$
- That $\log_a b^m = m\log_a b$

(15 minutes)

Step 6.3.2 : • Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers, pastels and calculators among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 6.3.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the characteristic of the logarithm of a number which is less than one is negative while the mantissa is positive
- That to represent the fact that the characteristic is negative, -2 for example is written as $\bar{2}$
- That the characteristic of the logarithm of a number between 0 and 1 is the number of zeros that appear before the first non-zero number on the right hand side of the decimal point, plus one
- That when a negative characteristic is multiplied by a number, the following steps need to be carried out

$$\bar{1}.8732 \times 3 = \bar{3} + 2.6196$$
$$= \bar{1}.6196$$

- That when a negative characteristic is being divided by a number, it has to be first re-organized so that the negative characteristic is divisible without remainder, and the following steps should to be carried out

$$\bar{1}.2732 \div 3 = \frac{\bar{3} + 2.2732}{3}$$
$$= \bar{1}.7577$$

(15 minutes)

Step 6.3.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(25 minutes)

Step 6.3.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That expressions which involve multiplication or division of numbers that are less than 1 can easily be simplified using the logarithmic tables
  - That special attention should be paid in carrying out simplifications when the characteristic of the logarithm is negative
    - $\bar{1}.8372 - \bar{2}.4372$
    - $\bar{1}.2372 - \bar{2}.4372$
  - That the solutions to problems obtained by using logarithms and by using a calculator are the same

(15 minutes)

Criteria for Assessment and Evaluation:

- Describes how the characteristic of the logarithm is obtained when finding the logarithm of a number which is less than 1.
- Accepts that complex multiplications and divisions can be done easily using logarithmic tables.
- Simplifies expressions involving the multiplication and division of several numbers including square roots and powers, using the logarithmic tables.
- Is inclined to check the accuracy of the results of an action by various methods.
- Shares experiences and knowledge between each other.

Annex 6.3.1

Instructions for group exploration

Let us facilitate simplification by using logarithmic tables

Part I

- Focus your attention on the following numbers which are all less than one.
  A = 0.527
  B = 0.0527
  C = 0.00527
  D = 0.000527
- Write the number that your group receives from the above numbers in scientific notation, and identify the characteristic of the logarithm of the number.
- Discuss how the characteristic of the logarithm of a decimal number less than 1 can be written without first writing the number in scientific notation.
- By means of it, write the logarithm of the given decimal number.
- Write down any decimal number less than 1 which you like and use the logarithmic tables to obtain its logarithm.
- By paying attention to the characteristic of the logarithm, find the square and the square root of the number using $\lg\sqrt{a} = \frac{1}{2}\lg a$ and $\lg a^2 = 2\lg a$.
- Confirm the accuracy of your results using a calculator.
- Prepare to creatively present your findings at the plenary session.

Part II

- Focus your attention on the following expressions.

Expression 1 : $\frac{a^2 \times \sqrt{b}}{c}$ Expression 2 : $\frac{a^3 \times \sqrt[3]{b}}{c}$

Expression 3 : $\frac{\sqrt{a} \times b^2}{c}$ Expression 4 : $\frac{\sqrt[3]{a} \times b^3}{c}$

- Substitute the values a = 0.6374, b = 0.0875 and c = 5.45 into the expression given to you.
- Write the logarithm of the expression.
- Find the value of the expression using the logarithmic tables, paying special attention to the characteristics of the logarithms.
- Confirm the accuracy of your result using a calculator.
- Prepare to present your findings at the plenary session.

04. Surface Areas of Solids

Competency 8 : Makes use of a limited space in an optimal manner by investigating the area.

Competency Level 8.1 : Investigates the surface areas of various solids in the surrounding.

Activity 8.1 : Let us find the surface areas of solids.

Time : 65 minutes.

Quality Inputs :
- Copies of the instruction leaflet on exploration included in Annex 8.1.1.
- Sets of two pyramids and two cones each with different bases and different altitudes and with nets that can be opened out.
- Gum and cello tape.
- Demy papers and pastels.

Learning – Teaching Process:

Step 8.1.1 :
- Inquire from the students how the area of a square, a triangle, a circle and a sector of a circle are obtained.
- Lead a discussion and highlight the following facts.

- That the area of a square with sides of length a units is a^2 square units
- That the area of a triangle is obtained by $\frac{1}{2} \times$ length of the base $\times$ altitude
- That the area of a circle of radius r is given by πr^2
- That the area of a sector of a circle with radius r and angle of the sector θ, is given by $\pi r^2 \times \frac{\theta}{360}$
- That the area of a sector of a circle can also be obtained by cutting the sector into thin triangular shaped strips and forming a rectangle

(15 minutes)

Step 8.1.2 : • Divide the class into small groups.
- Distribute the instructions on exploration, demy papers and pastels and the sets of pyramids and cones among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(25 minutes)

Step 8.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the surface area of a pyramid with a square base equals the sum of the area of the base and the areas of the four triangular faces
- That the surface area of a cone equals the sum of the area of the curved surface and the area of the base
- That the area of the curved surface of a cone equals the area of the rectangle with length equal to half the circumference of the circular base and breadth equal to the slant height of the cone
- That accordingly, the area of the curved surface of a cone with base radius r and slant height l is πrl
- That the surface area of a sphere of radius r is given by $4\pi r^2$

(25 minutes)

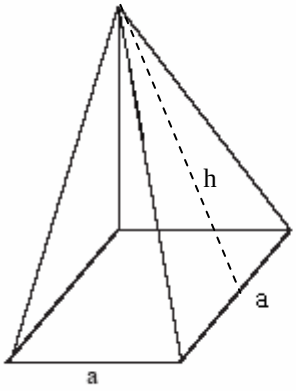
Criteria for Assessment and Evaluation:

- Describes the shapes of the various surfaces of pyramids, cones and spheres.
- Accepts that it is easy to find the surface areas of solids using formulae.
- Calculates the surface area when the data related to the solid is provided.
- Discovers facts through analysis.
- Simplifies a complex action by dividing it into parts.

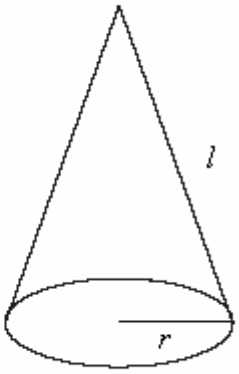
Annex 8.1.1

Instructions for group exploration

Let us find the surface areas of solids



Solid (1)



Solid (2)

- Focus your attention on the solid that your group received from the above solids.
- Obtain the measurements of the solid as expressed in the above figure.
- What is the name given to the solid?
- Open out the net of the solid and identify the shapes of the surface parts.
- Find the surface area of the solid by separately calculating the areas of all the surfaces of the solid, ignoring the allowance for pasting.
- Use this to develop a formula for the surface area of the solid you obtained in terms of the measurements (a, h / r, l) given in the figure.
- Prepare to explain to others how your group obtained the formula for the surface area of the solid.

05. Volumes of Solids

Competency 10 **:** Gets the maximum out of space by working critically with respect to volume.

Competency Level 10.1 **:** Explores the volumes of various solids.

Activity 10.1 **:** Let us find the volumes of solids.

Time **:** 110 minutes.

Quality Inputs **:**
- Model of a cube, a cuboid and a cylinder.
- A hollow cuboid and a hollow pyramid with identical square bases and of equal altitude.
- A hollow cylinder and a hollow cone with identical bases and of equal altitude.
- Sand.
- Copies of the instruction leaflet on exploration included in Annex 10.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 10.1.1 **:**
- Present the models of the cube, cuboid and cylinder to the class and lead a discussion on finding their volumes.
- During the discussion highlight the following facts.

  - That the volume of a cube with sides of length a is a^3
  - That the volume of a cuboid with length, width and height equal to a, b, c respectively is abc
  - That the volume of a right circular cylinder with base radius r and altitude h is $\pi r^2 h$

(10 minutes)

Step 10.1.2 **:**
- Divide the class into four small groups.
- Distribute the instructions on exploration, the solids relevant to each group, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (20 minutes)

Step 10.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the volume of a pyramid with base a square with sides of length a and of altitude h equals $\frac{1}{3}a^2h$
- That the volume of a cone with base radius r and altitude h is $\frac{1}{3}\pi r^2 h$
- That the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$

(30 minutes)

Step 10.1.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for presentations at the plenary session.

(20 minutes)

Step 10.1.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the volume of a cone with fixed base radius changes in proportion to the change in the altitude
- That when the altitude of a cone remains unchanged, the volume of the cone changes in proportion to the square of the change in the base radius

(30 minutes)

Criteria for Assessment and Evaluation:

- Describes how the volume of a cone and a pyramid are found by means of the volume of a cylinder and a cuboid.
- Accepts that it is easy to find the volumes of solids using formulae.
- Finds the volumes of solids using formulae.
- Makes decisions logically.
- Critically evaluates the constructive comments of other groups.

Annex 10.1.1

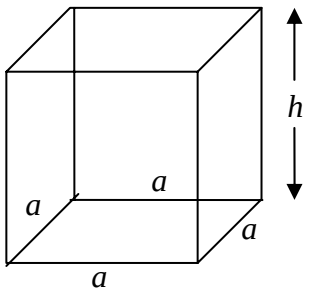
Instructions for group exploration

Let us find the volumes of solids

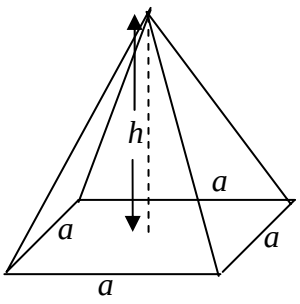
Part I

A

1

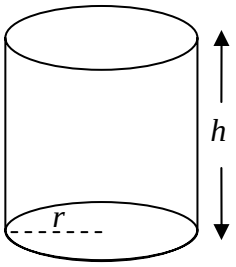


2

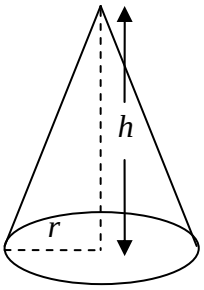


B

1



2



- Engage in the relevant activity using the set of solids obtained by your group from the above sets.
- Fill the second object with sand and pour it into the first object. Repeat the process until the first object is completely filled with sand.
- Hence find a relationship between the volumes of the two objects.
- Using the formula for the volume of the first object, develop a formula for the volume of the second object.
- Discuss whether a formula can be developed for the volume of a sphere of radius r.
- Prepare for a presentation at the plenary session.

Part II

Radius of the base of the cone	Altitude of the cone
i. 7cm	10cm
ii. 7cm	14cm
iii. 3.5cm	12cm

- Focus your attention on the measurements given to your group from the measurements of cones given above.
- Find the volume when the altitude remains constant but the base radius is doubled.
- Find the volume when the base radius remains the same but the altitude is doubled.
- Find the volume when the base radius remains the same but the altitude is halved.
- Discuss using the above results regarding the change in the volume when the altitude h of a cone of base radius r remains constant while the base radius is changed, and the change in the volume when the base radius remains the same while the altitude is changed.
- Prepare to present your results at the plenary session.

06. Binomial Expressions

Competency 14 : Simplifies algebraic expressions by systematically exploring various methods.

Competency Level 14.1 : Determines the cubes of binomial expressions.

Activity 14.1 : Let us find the cubes of binomial expressions.

Time : 90 minutes.

Quality Inputs :
- The figure included in Annex 14.1.1.
- Four copies of the instruction leaflet on exploration included in Annex 14.1.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 14.1.1 :
- Present the figure to the class.
- Lead a discussion and highlight the following facts.

• That the product of two binomial expressions, such as $(a+b)(c+d)$, can be obtained by taking the sum of the areas of several rectangles • That $(a+b)^3$ is the cube of $(a+b)$ • That the volume of a cube or cuboid is obtained by length × breadth × height

(15 minutes)

Step 14.1.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let them prepare for a presentation at the plenary session.

(30 minutes)

Step 14.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the cube $(a+b)^3$ may be written as $a^3+3a^2b+3ab^2+b^3$
- That when a binomial expression is cubed, the terms of the cubed expression are in a certain pattern
- That the cube of a binomial expression can easily be found using the volumes of the parts of a cube
- That the cube of (a – b) may be obtained by substituting (-b) for b

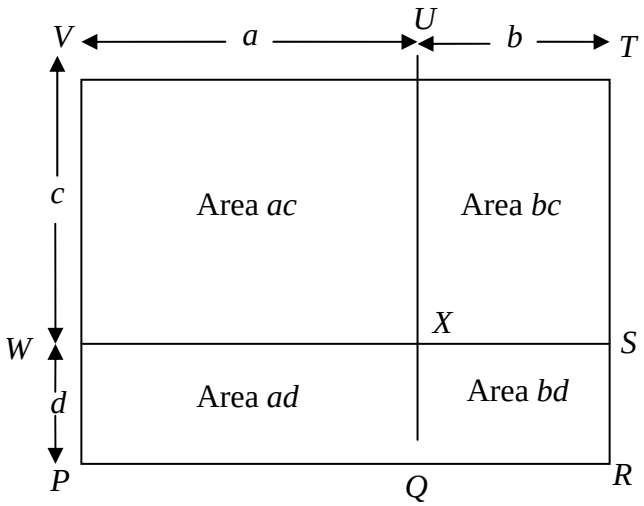
(45 minutes)

Criteria for Assessment and Evaluation:

- Expresses the pattern of the terms of the cube of a binomial expression.
- Accepts that the terms of the cube of a binomial expression can easily be obtained using the parts of a cube.
- Writes down the cubes of various binomial expressions.
- Presents ideas logically.
- Easily sees the whole by analysis.

Annex 14.1.1

Figure



$(a+b)(c+d) =$

Annex 14.1.2

Instructions for group exploration

Let us find the cubes of binomial expressions

Figure 1

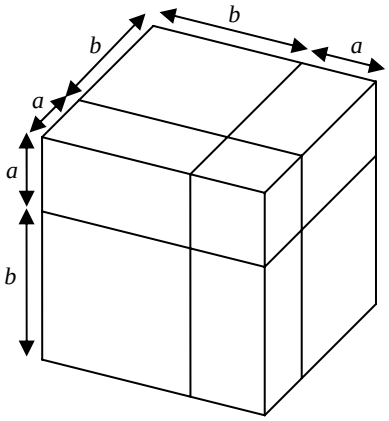


Figure 2

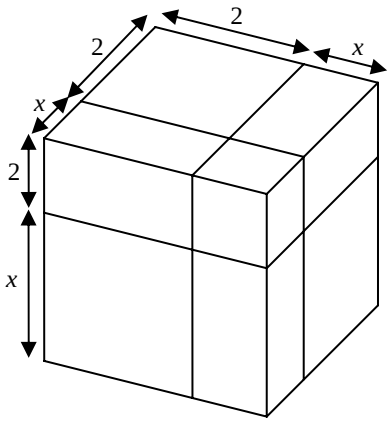


Figure 3

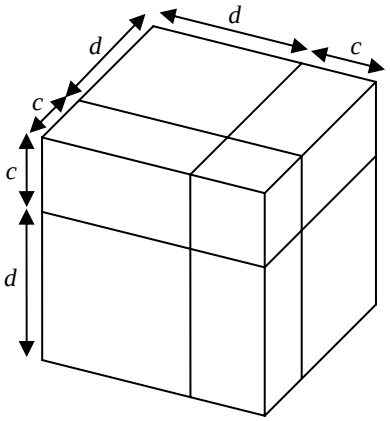
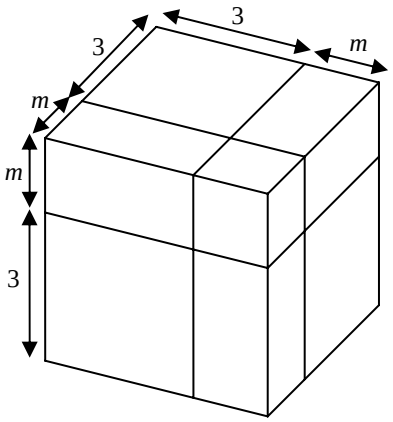


Figure 4



- Focus your attention on the figure of the cube assigned to your group from the above figures.
- Develop an expression for the volume of the cube using the given lengths of the sides.
- Develop a relationship between the volumes of the cubes and cuboids obtained by separating the cube into parts as shown in the figure, and the expression written for the volume of the whole cube.
- Propose another method to obtain the above relationship and determine the expansion of $(x+y)^3$.
- Use the above to find the expansion of $(x-y)^3$.
- Prepare for a presentation at the plenary session.

07. Algebraic Fractions

Competency 16 : Solves problems encountered in day to day life by exploring the various methods of simplifying algebraic fractions.

Competency Level 16.1 : Manipulates algebraic fractions under the basic mathematical operations.

Activity 16.1 : Let us simplify algebraic fractions.

Time : 120 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 16.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 16.1.1 :
- Inquire from the students how two simple algebraic fractions are added and subtracted, and also how two fractions are multiplied and divided.
- Lead a discussion and highlight the following facts.

  - That when adding and subtracting algebraic fractions, the denominators have to be made equal
  - That when multiplying normal fractions, the numerator and denominator can be divided by the same whole number
  - That when dividing a normal fraction by another fraction, instead of dividing, it is multiplied by the reciprocal of the divisor

(15 minutes)

Step 16.1.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let them prepare for a presentation at the plenary session.

(20 minutes)

Step 16.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.

- Engage in a review so that the following facts are highlighted.

  - That when multiplying algebraic fractions, the numerator should be multiplied by the numerator and the denominator by the denominator
  - That when multiplying, the denominator and numerator can be divided by the same value
  - That when dividing algebraic fractions, instead of dividing by the divisor, the dividend can be multiplied by the reciprocal of the divisor
  - That the final solution should be given in the most simplified form
  - That multiplication and division of algebraic fractions can be carried out in the same manner as multiplication and division of normal fractions

(30 minutes)

Step 16.1.4 **:**

- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let them prepare for a presentation at the plenary session.

(25 minutes)

Step 16.1.5 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That the above method may also be used to multiply and divide algebraic fractions with complex expressions
  - That by finding the factors of the algebraic expressions with factors, the most simplified form can be obtained

(30 minutes)

Criteria for Assessment and Evaluation:

- Describes how algebraic fractions are simplified under the mathematical operations.
- Accepts that the process used to simplify normal fractions can be used for algebraic fractions too.
- Simplifies two algebraic fractions under a binary mathematical operation.
- Obtains the solution to a problem easily by following steps.
- Is inclined to engage in logical discussions within the group when solving problems.

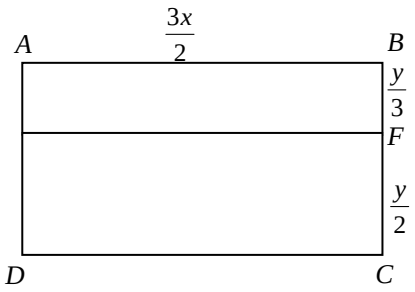
Annex 16.1.1

Instructions for group exploration

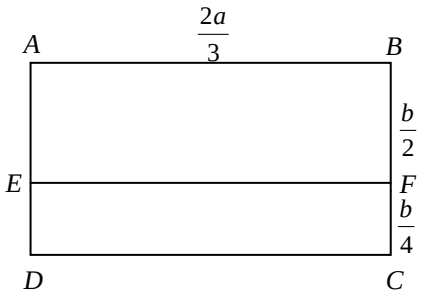
Let us simplify algebraic fractions

Part I

(1)



(2)



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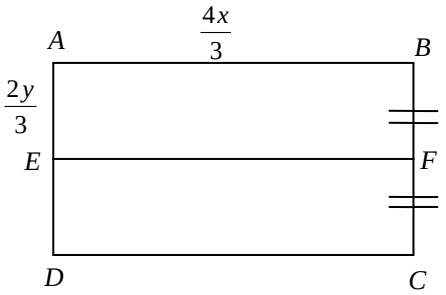

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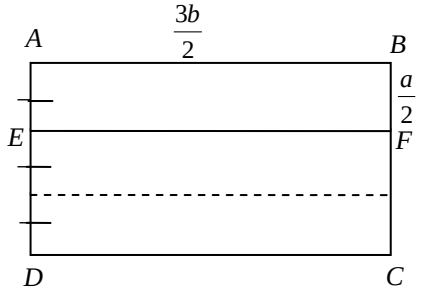


- Carefully examine the rectangle *ABCD* obtained by your group from the rectangles that are given above.
- Find the area of the rectangle *ABCD* by obtaining its length and breadth.
- Obtain the ratio between the areas of the rectangle *ABFE* and the rectangle *ABCD*.
- Prepare to present the facts that you found out about multiplying and dividing algebraic fractions using the above obtained results.

Part II

(A)

i. $\frac{2x}{3(x+y)} \times \frac{6(x+y)}{5y}$

ii. $\frac{2x}{3(x+y)} \div \frac{6y}{7(x+y)}$

iii. $\frac{5}{a^2-b^2} \div \frac{a+b}{a-b}$

(B)

i. $\frac{3a}{2(2a+b)} \times \frac{6(2a+b)}{5b}$

ii. $\frac{4a}{3(a-2b)} \div \frac{3b}{5(a-2b)}$

iii. $\frac{x^2-y^2}{2x} \div \frac{x-y}{x+y}$

(C)

i. $\frac{4a}{5(3a-b)} \times \frac{7(3a-b)}{5b}$

ii. $\frac{2x}{5(2x-3y)} \div \frac{3y}{7(2x-3y)}$

iii. $\frac{3(a^2-9b^2)}{5b} \div \frac{a-3b}{a+3b}$

(D)

i. $\frac{4xy}{7(2x-y)} \times \frac{2(2x-y)}{3y}$

ii. $\frac{2a}{3a-2b} \div \frac{5b}{2(3a-2b)}$

iii. $\frac{3a}{4a^2-b^2} \div \frac{2a-b}{2a+b}$

- Focus your attention on the problems that your group received from the above sets of problems.
- Discuss the steps that are required to simplify them.
- Prepare to present how the simplification is done, at the plenary session.

08. The Areas of Rectilinear Plane Figures between Parallel Lines I

Competency 23 : Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency Level 23.1 :Determines the relationship between the areas of parallelograms and triangles that lie between the same parallel lines.

Activity 23.1 : Let us compare the areas of parallelograms and triangles that lie between the same pair of parallel lines.

Time : 95 minutes.

Quality Inputs :
- The figure included in Annex 23.1.1.
- Three copies of the instruction leaflet on exploration included in Annex 23.1.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 23.1.1 :
- Present the chart of figures to the class and lead a discussion on the areas of the figures.
- During the discussion highlight the following facts.

• That the perpendicular distance from any point on a straight line to a line parallel to it is a constant • That the area of a triangle of base length a and altitude h is $\frac{1}{2}ah$ • That the area of a parallelogram of base length a and altitude h, equals ah

(10 minutes)

Step 23.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(25 minutes)

Step 23.1.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the areas of parallelograms on the same base and between the same parallel lines are equal • That the accuracy of the above theorem can be verified when measurements are given • That the above theorem can be proved • That problems can be solved using the above theorem

(20 minutes)

Step 23.1.4 **:**

- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 23.1.5 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That if a parallelogram and a triangle stand on the same base and between the same two parallel lines, then the area of the triangle is half the area of the parallelogram • That the accuracy of the above theorem can be verified when measurements are given • That the above theorem can be verified using areas • That the above theorem can be used to solve problems

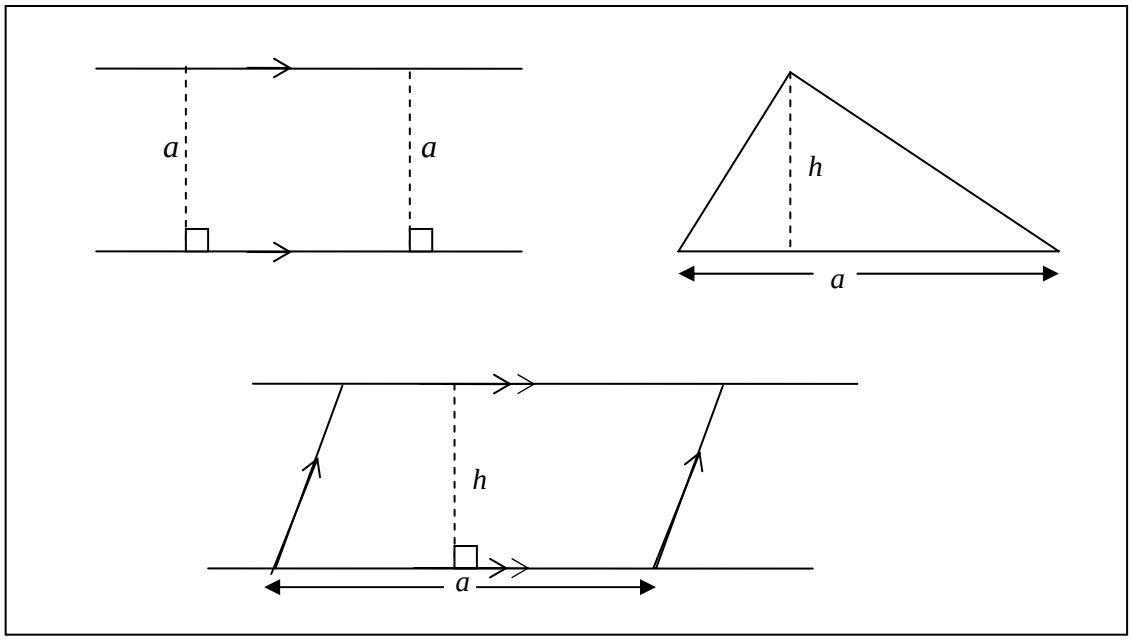
(20 minutes)

Criteria for Assessment and Evaluation:

- Describes the relationship between the area of a parallelogram and the area of a triangle on the same base and between the same two parallel lines.
- Accepts that congruent plane figures are not only equal in shape but also equal in area.
- Correctly manipulates measurements to examine the accuracy of a relationship.
- Makes right decisions by basing activities on facts.
- Expresses ones ideas logically.

Annex 23.1.1

Chart of Figures



Annex 23.1.2

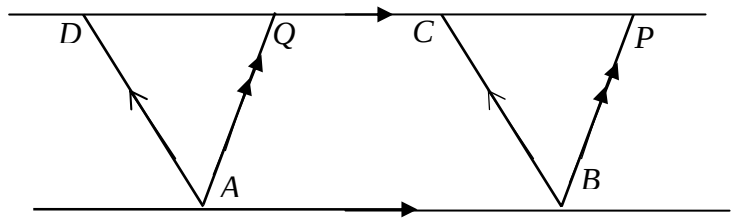
Instructions for group exploration

Let us compare the areas of parallelograms and triangles that lie between the same pair of parallel lines

Part I

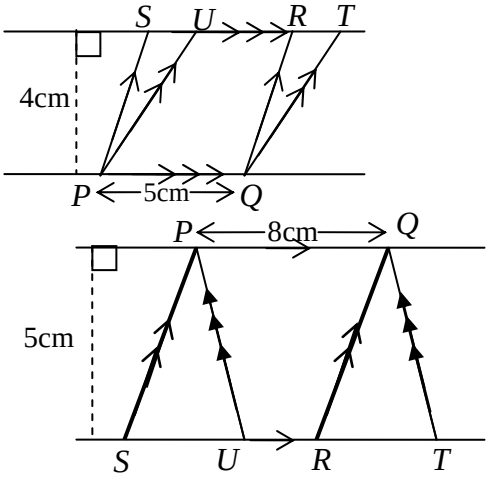
- Study the following chart.

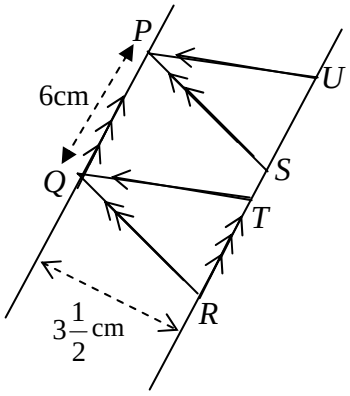
Theorem: ***Parallelograms on the same base and between the same parallel lines are equal in area.***



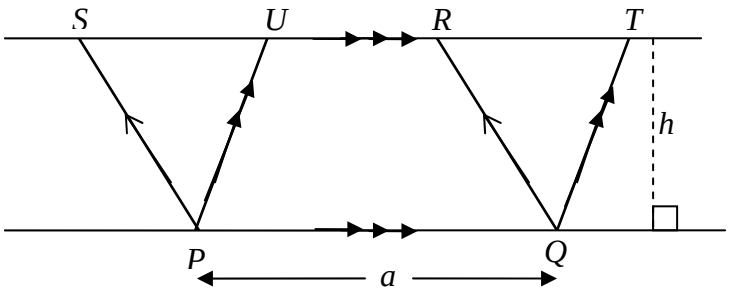
Area of the parallelogram *ABCD* = The area of the parallelogram *ABPQ*

- Focus your attention on the figure received by your group from the figures given below.





- Find the areas of the parallelograms *PQRS* and *PQTU* by paying attention to their measurements.
- Using the areas that you obtained, examine the accuracy of the theorem that you studied above.

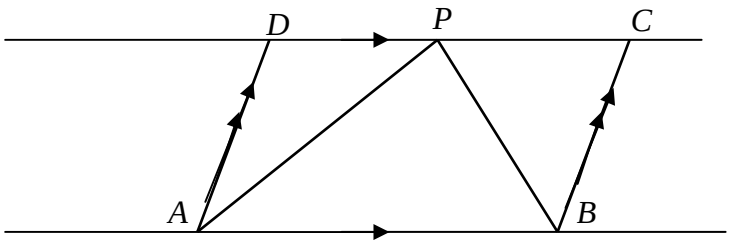


- Study the two parallelograms contained in this figure and show that the above theorem is true (using congruency of triangles).
- Prepare to present the group's findings at the plenary session.

Part II

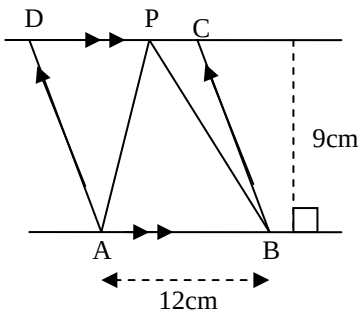
- Study the following theorem.

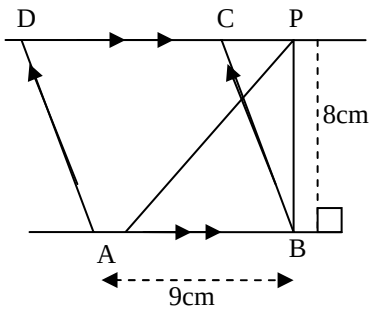
- Theorem: ***If a parallelogram and a triangle stand on the same base and are between the same two parallel lines, then the area of the triangle is half the area of the parallelogram.***

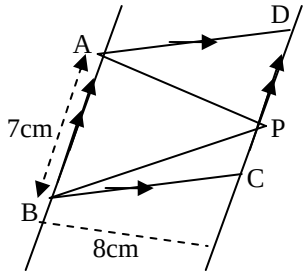


Area of the triangle $ABP = \frac{1}{2}$ the area of the parallelogram $ABCD$.

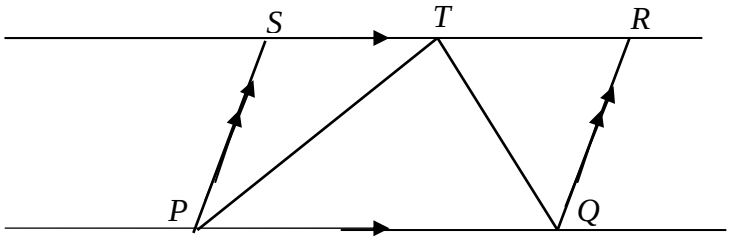
- Focus your attention on the figure received by your group from the figures given below.







- Using the measurements of the parallelogram and the triangle obtain a relationship between their areas.
- Examine the truth/falsehood of the theorem you studied above.



- Propose a method to show that the theorem is true by using the above diagram and drawing a line parallel to SP through the point T.
- Prepare to present to the other students how the theorem can be logically shown to be true using the above method.

08. The Areas of Rectilinear Plane Figures Between Parallel Lines II

Competency 23 : Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency Level 23.2 : Uses the relationship between the areas of triangles with common bases and common vertices to make decisions.

Activity 23.2 : Let us identify the theorems related to the areas of triangles.

Time : 95 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 23.2.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 23.2.1 :
- Inquire from the students about parallel lines, about the area of a triangle with base length a and altitude h and about the simplest form of ratios such as 12:8, $4x : 6x$, $\frac{1}{2}ax : 6\frac{1}{2}bx$.
- Lead a discussion and highlight the following facts.

> - That the gap between two parallel lines is equal
> - That the area of a triangle with base length a and altitude h is $\frac{1}{2}ah$
> - That a ratio could be given in the most simplified form by dividing the terms of the ratio by their greatest common divisor

(10 minutes)

Step 23.2.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (20 minutes)

Step 23.2.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.

- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the areas of triangles on the same base and between the same pair of parallel lines are equal • That the above theorem can be verified when measurements are given • That the above theorem can be proved for the general case • That this theorem can be used to solve problems

(25 minutes)

Step 23.2.4 **:**
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 23.2.5 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the areas of triangles of equal altitude are proportional to one another as their bases • That the above theorem can be verified when measurements are given • That the above theorem can be proved for the general case • That the above theorem can be used to solve problems

(20 minutes)

Criteria for Assessment and Evaluation:

- Describes the common features of triangles of equal altitude and bases on the same straight line.
- Accepts that geometric theorems can be verified by obtaining the required measurements and correctly manipulating them.
- Manipulates data to examine the truth/falsehood of given laws.
- Logically examines facts and generalizes other opinions.
- Engages in learning by applying one's knowledge.

Annex 23.2.1

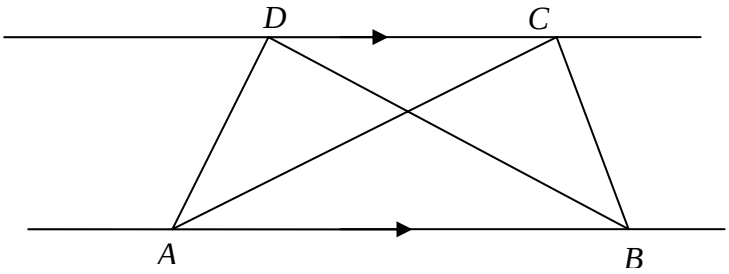
Instructions for group exploration

Let us identify the theorems related to the areas of triangles

Part I

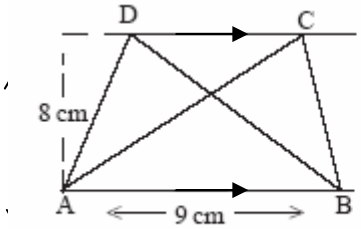
- Study the following theorem.

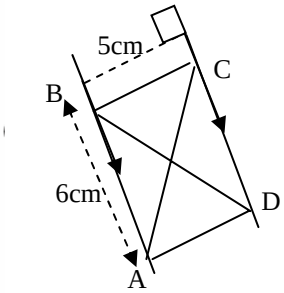
Theorem: ***Triangles on the same base and between the same parallel lines are equal in area.***

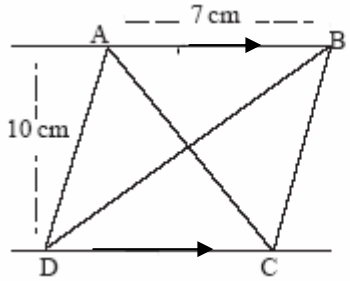


Area of the triangle *ABC* = Area of the triangle *ABD*

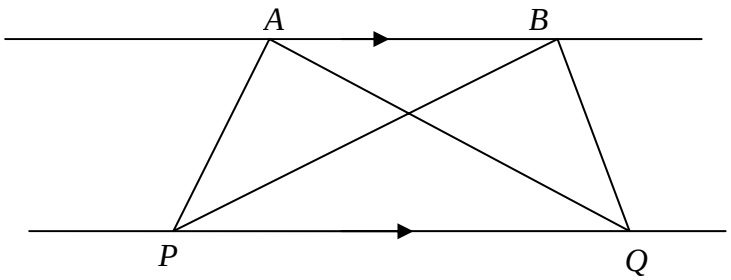
- Study the figure obtained by your group from the figures given below.







- Confirm that the above theorem is correct using the given information.
- Describe the procedure that you followed to confirm the theorem.

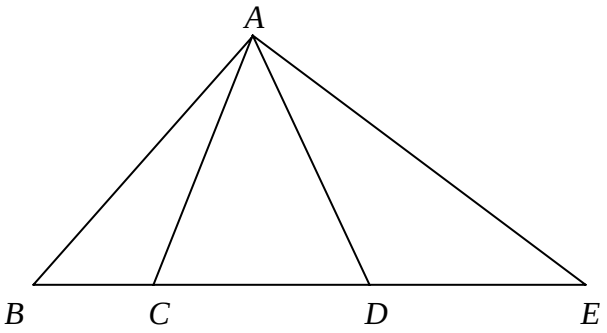


- Discuss how you can show that the above theorem is true by completing the parallelograms related to the triangles in the above figure.
- Discuss how the above theorem can be used to construct a triangle that has the same area as a given triangle.
- Prepare to present your group's findings to the others.

Part II

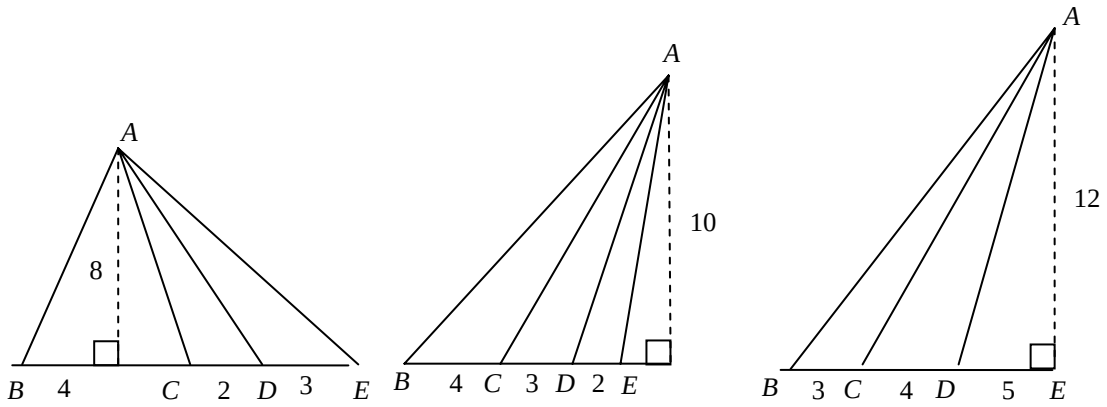
- Study the theorem given below.

Theorem: ***The areas of triangles of equal altitude are proportional to one another as their bases.***



Area of ΔABC : Area of $\Delta ADE = BC : DE$

- Focus your attention on the figure received by your group from the figures given below.



- All the measurements in the above figure are in cm.
- Using the given measurements write the ratio of the area of ΔABC : the area of ΔADE in the simplest form.
- Obtain the relationship between the ratio BC : DE and the ratio obtained above.
- In the above manner, determine the ratios for any two other triangles in the figure and obtain the relationship between them.
- Examine the accuracy of the theorem using the obtained results.

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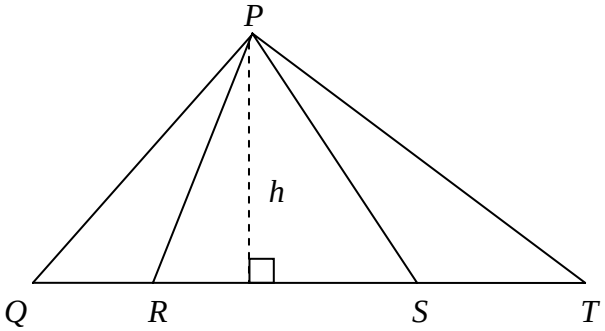


- Obtain the relationship for the general case in the above figure too.
- Discuss how your findings can be used to divide any triangle into several triangles of equal area.
- Prepare to explain the group's findings to the others.

09. Percentages I

Competency 5 : Makes successful transactions in the modern world of commerce by using percentages.

Competency Level 5.1 : Uses percentages when making transactions in installments.

Activity 5.1 : Let us calculate the interest on the reducing balance.

Time : 90 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 5.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 5.1.1 :
- Present the following problem to the class; 'Find the total amount payable on a loan of Rs. 5000 borrowed at a simple interest rate of 36% per annum, if the loan is to be repaid in 6 months'.
- Lead a discussion and highlight the following facts.

• That when the interest rate per annum and the principal amount are given, the monthly interest can be calculated • That the total amount payable is obtained by adding the interest to the principal amount • That the sum of the first n positive integers equals $\frac{n}{2}(n+1)$

(20 minutes)

Step 5.1.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the same task to two groups.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(40 minutes)

Step 5.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That in hire purchases the interest is calculated on the reducing balance
- That the total interest that is paid in hire purchases can be obtained as the product of the interest on the monthly portion of the loan and the number of month units
- That the number of month units = $\frac{1}{2}$ (number of installments)(number of installments + 1)
- That the interest calculated on the reducing balance is less than the interest calculated according to the usual method
- That there are advantages as well as disadvantages in hire purchases

(30 minutes)

Criteria for Assessment and Evaluation:

- Describes the nature of hire purchases.
- Accepts that there are advantages as well as disadvantages in making payments in installments.
- Calculates the total interest payable on a hire purchase.
- Does transactions in the world of commerce with greater thought.
- Is more attentive when making purchases.

Annex 5.1.1

Instructions for group exploration

Let us calculate the interest on the reducing balance

- Focus your attention on the case that is assigned to your group from the following cases.

Case I	Case 2
Payment in 8 installments of Rs. 3000 excluding interest, for an item. Monthly interest rate of 5%	Payment in 8 installments of Rs. 2000 excluding interest, for a loan. Monthly interest rate of 6%

- Since the amount has to be paid in 8 installments, under the reducing balance scheme, in the first month, interest has to be paid for the 8 installments. In the second month, interest has to be paid for 7 installments. By studying the textbook find out facts about this.
- Find the total number of installments on which interest has to be paid by considering the number of installments on which interest has to be paid each month.
- Develop a formula to find the total number of installments by using your knowledge on progressions.
- Now find the amount of interest that should be paid on one installment for a month, and use this to find the interest payable for the total number of installments you obtained above.
- If payment is made in equal installments determine how much an installment payment is with the interest.
- Find the interest that should be paid in 8 months for the amount given to your group and for the given interest rate, calculated according to the normal method, and compare it with the amount you obtained by calculating the interest on the reducing balance.
- Discuss the reasons for the difference in the two interests.
- Prepare for a creative presentation at the plenary session.

09. Percentages II

Competency 5 : Makes successful transactions in the modern world of commerce by using percentages.

Competency Level 5.2 : Makes transactions by comparing schemes of interest.

Activity 5.2 : Let us identify the more beneficial interest scheme.

Time : 90 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 5.2.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 5.2.1 :
- Write the following on the blackboard and get the ideas of the students about it; 'Simple interest is paid at a rate of 14% per annum for fixed deposits'.
- Lead a discussion and highlight the following facts.

  - That in loan transactions as well as in making deposits, the amount paid apart from the principal amount is the interest
  - That it should be decided initially whether the interest is paid monthly, quarterly, semi-annually or annually
  - That when the interest paid is the same for each installment, the interest is simple interest
  - That to be free from a loan, the loan amount as well as the interest calculated for the time period of the loan should be paid

(15 minutes)

Step 5.2.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each groups.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 5.2.3 :
- Provide each small group with the opportunity to present the findings of the group.

- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That if the interest obtained from the first installment is added to the principal amount and this is considered as the principal amount for the next installment, the interest is compound interest • That the compound interest calculated on a certain amount is more than the simple interest on the same amount, calculated with the same interest rate and for the same period of time

(15 minutes)

Step 5.2.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(15 minutes)

Step 5.2.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That for deposits, compound interest is more profitable for the depositor than simple interest • That for loan transactions, compound interest is disadvantageous for the borrower • That for bank deposits, compound interest is calculated daily and added to the principal amount monthly

(15 minutes)

Criteria for Assessment and Evaluation:

- Describes the advantages and disadvantages between simple interest and compound interest schemes.
- Accepts that compound interest schemes are advantageous for depositors and disadvantageous for borrowers.
- Calculates the interest and the total value of an investment or loan under simple interest as well as compound interest schemes.
- Has an awareness of the transactions carried out in the surroundings.
- Engages in learning by analyzing information.

Annex 5.2.1

Instructions for group exploration

Let us identify the more beneficial interest scheme

Part I

- Focus your attention on the method that is assigned to your group from the following methods of paying interest.
  A - Monthly
  B – Quarterly
  C – Semi-annually
  D – Annually

- Read from your textbook the report issued to a depositor by a principal establishment regarding deposits.
- Considering the interest scheme given in the report, find the total value after 3 installments, of an investment of Rs. 100 000, if the annual interest rate is 12% and interest is paid according to the method assigned to your group.
- Find the total value of the above investment after 3 installments if simple interest is paid at an annual rate of 12% according the method assigned to your group.
- Discuss the similarities and the differences between the total values of the investment under the two schemes.
- By considering its properties, propose a name for the new interest scheme that you learnt about and prepare a report on it that includes these properties.
- Prepare to creatively present the findings of your group at the plenary session.

Part II

- Focus your attention on the event that your group received from the events given below.
  - A deposit of Rs. 20 000 in a bank that pays interest monthly at an annual interest rate of 12%.
  - A deposit of Rs. 20 000 in a bank that pays interest monthly at an annual interest rate of 24%.
  - A loan of Rs. 20 000 from a bank that charges interest monthly at an annual interest rate of 12%.
  - A loan of Rs. 20 000 from a bank that charges interest monthly at an annual interest rate of 24%.
- Find the total value of the deposit/loan in the event your group received, after three installments of compound interest.
- If the event that your group received was changed from a deposit to a loan, or from a loan to a deposit, what can you say about the amount that you would have to pay or you would receive?
- Prepare a report describing the advantages and disadvantages for the consumer, of compound interest on a loan or a deposit.
- Discuss and collect information on the way interest is calculated for bank deposits.
- Prepare to creatively present your group's findings at the plenary session.

10. Businesses and Shares

Competency 5 : Makes successful transactions in the modern world of commerce by using percentages.

Competency Level 5.3 : Considers the share market to invest money.

Activity 5.3 : Let us make investments with awareness.

Time : 60 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 5.3.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 5.3.1 :
- Begin the lesson by inquiring from the students about safe and profitable methods of depositing money that has been earned.
- Inquire whether there are other profitable methods of investing money, as the compound interest scheme for fixed deposits.
- Lead a discussion and highlight the following facts.

  - That a person with money has methods of making an income from that money
  - That an income can be obtained from a compound interest scheme by depositing money in a bank
  - That an income can also be made from fixed deposits
  - That there are other methods of making a profit by depositing money
  - That investing money in a business is another method of making a profit

(15 minutes)

Step 5.3.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 5.3.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That in calculations related to joint businesses, terms such as capital, primary issue, nominal value of a share, commercial value and dividend are used
- That when shares are bought at a lower value than the nominal value the value of the investment in the company is more, and when shares are bought at a higher value than the nominal value, the value of the investment is less than what was invested
- That the profit from an investment can be calculated if the amount invested, the commercial value of a share, the nominal value of a share, dividend and the period of investment are given
- That by comparing the advantages and disadvantages, the most suitable business to invest money in could be selected

(15 minutes)

Criteria for Assessment and Evaluation:

- Describes the terms related to joint businesses.
- Decides that it is better to invest money in a profitable business.
- Calculates the dividend for various businesses using percentages.
- Critically considers the different opinions regarding investing money in a business.
- Manipulates money in a beneficial way in daily activities.

Annex 5.3.1

Instructions for group exploration

Let us make investments with awareness

- Focus your attention on the company that your group received from the following companies.

	Name of the Company	Commercial Value	Nominal Value	Dividend
1. Lower Price	A	6	10	10%
2. Same Price	B	10	10	10%
3. Higher Price	C	15	10	10%

- Collect information about investing money in businesses by reading your textbook.
- Find the profit when Rs. 45 000 is invested for a year in the company your group received.
- Find the interest obtained after a year, on a fixed deposit of Rs. 45 000 placed in a bank at an interest rate of 10% per annum.
- If there is a difference between the profit obtained from the company and the interest obtained from the bank, give reasons for it.
- Discuss what the dividend should be if you are to obtain a profit from a company by purchasing shares at a higher price.
- Prepare to present your group's findings creatively at the plenary session.

11. Mid-point Theorem

Competency 23 : Makes decisions regarding day to day activities based on geometric concepts related to rectilinear plane figures.

Competency Level 23.6 : Examines the results when the sides of a triangle are divided proportionally.

Activity 23.6 : Let us verify the mid-point theorem.

Time : 120 minutes.

Quality Inputs :
- An enlarged copy of the Chart of Figures included in Annex 23.6.1
- Three copies of the instruction leaflet on exploration included in Annex 23.6.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 23.6.1 :
- Present the chart of figures to the class and inquire from the students about the properties of parallelograms, alternate angles and instances of congruency of triangles.
- Lead a discussion and highlight the following facts.
  - That pairs of opposite sides of a parallelogram are parallel
  - That a quadrilateral in which a pair of opposite sides is equal in length and parallel is a parallelogram
  - That the alternate angles formed when a transversal cuts a pair of parallel lines are equal
  - That there are four instances of congruency of triangles

(15 minutes)

Step 23.6.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 23.6.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the straight line joining the mid-points of two sides of a triangle is parallel to the third side • That the length of the line segment joining the centers of two sides of a triangle is half the length of the third side

(30 minutes)

Step 23.6.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 23.6.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the straight line drawn from the mid-point of one side of a triangle and parallel to another side, bisects the third side • That this is defined as the converse of the mid-point theorem

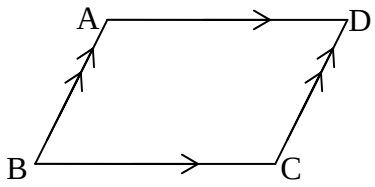
(15 minutes)

Criteria for Assessment and Evaluation:

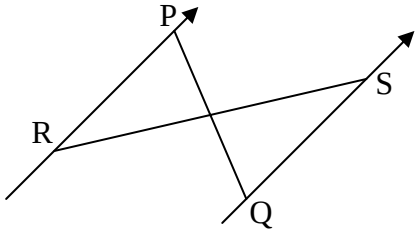
- Expresses the mid-point theorem and its converse.
- Accepts that the mid-point theorem and its converse are true.
- Proves the mid-point theorem and its converse.
- Methodically verifies facts.
- Makes decisions logically.

Annex 23.6.1

Chart of Figures



AB = DC, AB//DC
AD = BC, AD//BC



$R\hat{P}Q = P\hat{Q}S$
$P\hat{R}S = R\hat{S}Q$

Annex 23.6.2

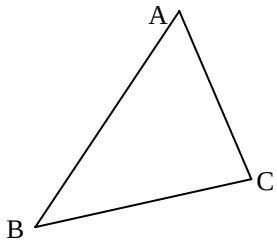
Instructions for group exploration

Let us verify the mid-point theorem

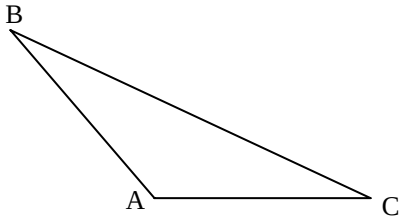
Part I

Theorem: ***The straight line joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.***

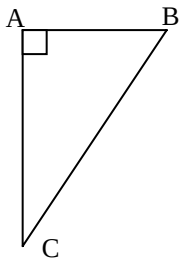
- Focus your attention on the triangle obtained by your group from the following triangles.



i. Acute angled triangle



ii. Obtuse angled triangle



iii. Right angled triangle

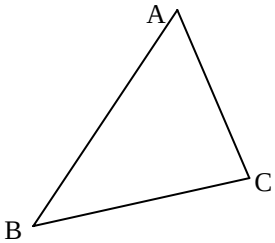
- Make an enlarged copy of the triangle and mark the mid-points of AB and AC as D and E respectively.
- Use a suitable method to verify the first part of the theorem and verify the second part of the theorem by measuring the appropriate lengths.
- Draw a line parallel to AB through C.
- Name the point of intersection of this parallel line and DE extended as F.
- Show that the triangles AED and CEF are congruent and hence discuss a method to show that the theorem is true for the general case.
- Prepare to creatively present your findings to the class.

Part II

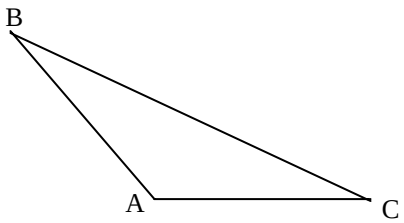
The converse of the mid-point theorem

The converse: ***The straight line through the mid-point of one side of a triangle and parallel to another side, bisects the third side***

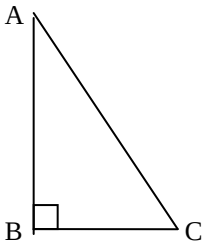
- Focus your attention on the triangle obtained by your group from the following triangles.



i. Acute angled triangle



ii. Obtuse angled triangle



iii. Right angled triangle

- Find the mid-point of AB and name it D.
- Construct a straight line parallel to AC through D.
- Investigate the truth of the above given converse.
- Discuss within the group a method to show the accuracy of the converse for the general case using the same construction that was used to prove the theorem.
- Prepare to creatively present your findings to the class.

12. Equi-angular triangles I

Competency 23 **:** Makes decisions regarding day to day activities based on geometric concepts related to rectilinear plane figures.

Competency Level 23.3 **:** Investigates the relationship between the sides of a triangle and parallelism.

Activity 23.3 **:** Let us find the relationship between the sides of a triangle and parallelism.

Time **:** 120 minutes.

Quality Inputs **:**
- Figure included in Annex 23.3.1
- Four copies of the instruction leaflet on exploration included in Annex 23.3.2.
- Demy papers and pastels.
- Set of mathematical instruments.
- Set of triangles mentioned in the leaflet on exploration.

Learning – Teaching Process:

Step 23.3.1 **:**
- Present the chart of figures to the students and inquire about the properties of the mid-point theorem.
- Lead a discussion and highlight the following facts.

> - That the straight line joining the mid-points of two sides of a triangle is parallel to the third side and equals half of it
> - That the line segments formed by the mid-points of the two sides with the mid-points are in the ratio 1:1

(20 minutes)

Step 23.3.2 **:**
- Divide the class into four small groups.
- Distribute the instructions on exploration, the 4 enlarged triangles as mentioned in the leaflet of exploration, mathematical instruments, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 23.3.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That a line drawn parallel to one side of a triangle cuts the other two sides proportionally • That the above theorem can be proved using the theorem that the areas of triangles of equal altitude are proportional to one another as their bases and the theorem that triangles on the same base and between the same parallel lines are equal in area • That the above relationship can be verified

(20 minutes)

Step 23.3.4 : • Organize the groups again.
- Distribute the instructions on exploration and the 4 triangles prepared according to the measurements given in the second part of the leaflet on exploration, mathematical instruments, demy papers and pastels among the groups.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for presentations at the plenary session.

(30 minutes)

Step 23.3.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That if a line cuts two sides of a triangle proportionally, it is parallel to the third side • That this relationship can be verified • That the converse of the theorem can be proved using the same theorems that were used to prove the theorem

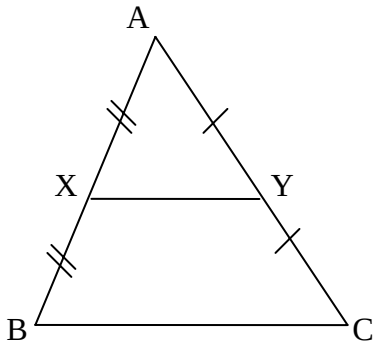
(20 minutes)

Criteria for Assessment and Evaluation:

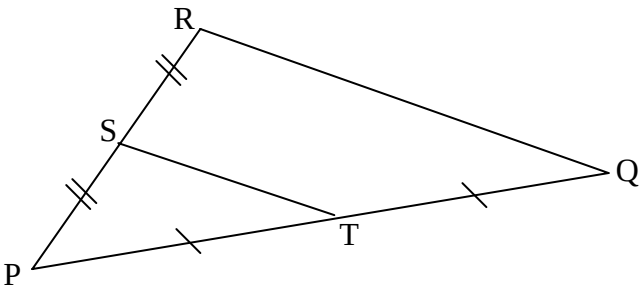
- Expresses the relationship between the sides of a triangle and parallelism.
- Accepts that by considering the relationship between the sides of two triangles, the special properties of two triangles can be discovered.
- Verifies the relationship between the sides of a triangle and parallelism.
- Does work according to standard methods.
- Makes decisions logically.

Annex 23.3.1

Chart of Figures



AX = XB
AY = YC



PT = TQ
PS = SR

Annex 23.3.2

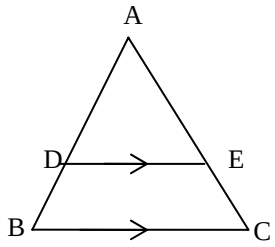
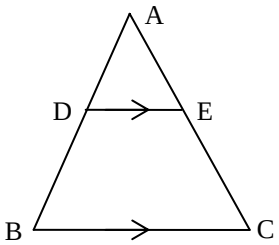
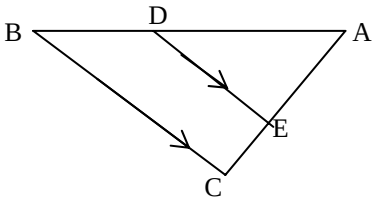
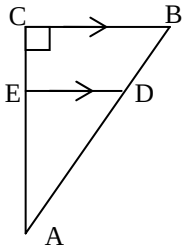
Instructions for group exploration

Let us find the relationship between the sides of a triangle and parallelism

Part I

Theorem: ***The straight line drawn parallel to one side of a triangle cuts the other two sides proportionally***

- Focus your attention on the triangle obtained by your group from the following triangles.

Triangle I  The equilateral triangle ABC	Triangle 2  The isosceles triangle ABC
Triangle 3  The obtuse angled triangle ABC	Triangle 4  The right-angled triangle ABC

- What is the line that that has been drawn parallel to one side of the large triangle?
- Determine the truth/falsehood of the above theorem by measuring the lengths of the two sections that the other two sides of the triangle are divided into by this parallel line intersecting them.
- Prove the theorem by considering the triangles BED and DEC that are formed by joining the points B and E and the points D and C, and the ratio of the areas of the triangles CDE and EDA.
- Write down all the theorems that you needed to use in this proof.
- Prepare to make a creative presentation at the plenary session.

Part II

The converse: ***If a line cuts two sides of a triangle proportionally, it is parallel to the third side***

- Focus your attention on the plane figure obtained by your group from the following plane figures.

Figure 1

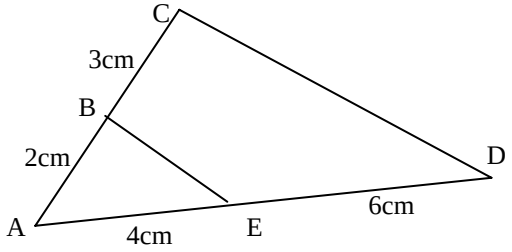


Figure 3

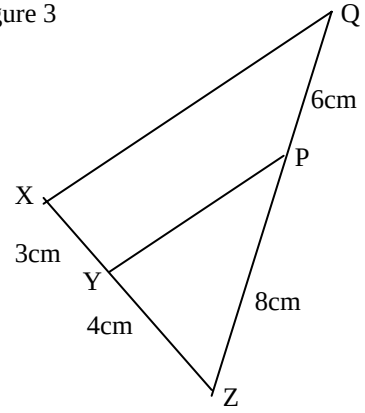


Figure 2

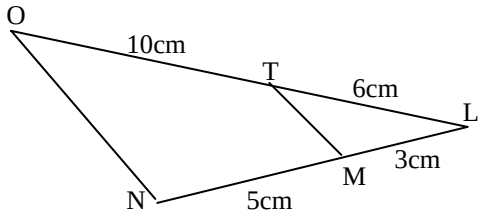
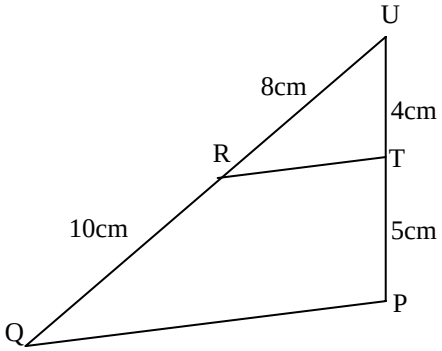


Figure 4



- Determine whether the ratios that the two sides of the larger triangle are divided into by the straight line that intersects them, are equal or not.
- Write down whether the converse is true/false by using a protractor to find the values of all the angles and checking for parallelism.
- Discuss how the converse can be proved in a manner similar to the way the theorem was proved by using the areas of triangles.
- Creatively prepare for a presentation at the plenary session.

12. Equi-angular triangles II

Competency 23 : Makes decisions regarding day to day activities based on geometric concepts related to rectilinear plane figures.

Competency Level 23.4 : Investigates the equi-angularity of triangles.

Activity 23.4 : Let us identify the properties of equi-angular triangles.

Time : 90 minutes.

Quality Inputs :
- An enlarged copy of the Chart of Figures included in Annex 23.4.1.
- Four copies of the instruction leaflet on exploration included in Annex 23.4.2. and a set of triangles with the measurements mentioned therein.
- Demy papers and pastels.

Learning – Teaching Process:

Step 23.4.1 :
- Present the chart of figures to the students.
- Lead a discussion and highlight the following facts.
  - That the corresponding sides as well as the corresponding angles of pairs of congruent triangles are equal
  - That the angles of a pair of triangles which are not congruent could also be equal

(10 minutes)

Step 23.4.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, the set of triangles with the measurements mentioned in the leaflet on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 23.4.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That if each of the angles of one triangle are equal to each of the angles of another triangle, the two triangles are said to be equi-angular
- That for two triangles to be equi-angular, the 3 angles of one triangle have to be equal to the three angles of the other triangle
- That although it is not necessary for equi-angular triangles to always be congruent, congruent triangles are always equi-angular
- That the corresponding sides of two equi-angular triangles are proportional
- That this theorem can be proved by using the theorem proved above

(20 minutes)

Step 23.4.4 **:**
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 23.4.5 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That if the corresponding sides of two triangles are proportional, they are equi-angular
- That this relationship can be used to solve problems

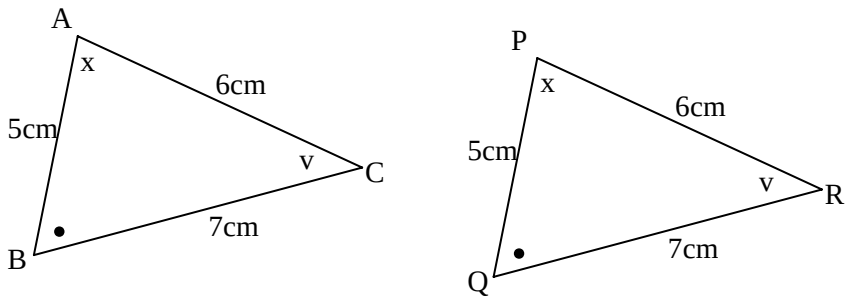
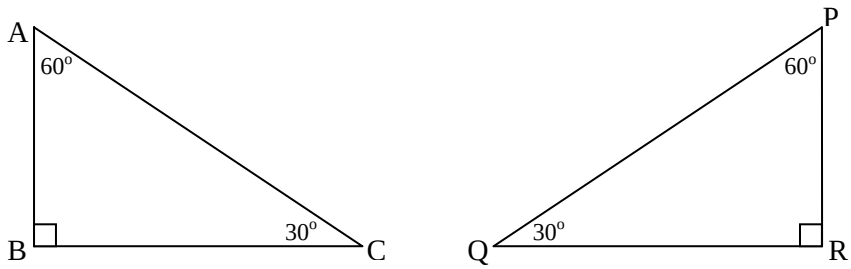
(20 minutes)

Criteria for Assessment and Evaluation:

- Expresses the theorem and the converse regarding the ratios of the sides of two equi-angular triangles.
- Accepts that two triangles being similar in shape, does not imply that they are congruent.
- Uses the theorem on equi-angular triangles to solve problems.
- Works as a group according to a set of instructions.
- Does not base decisions on observations alone, but makes logical decisions by further examination.

Annex 23.4.1

Chart of Figures

Case I	
Case II	

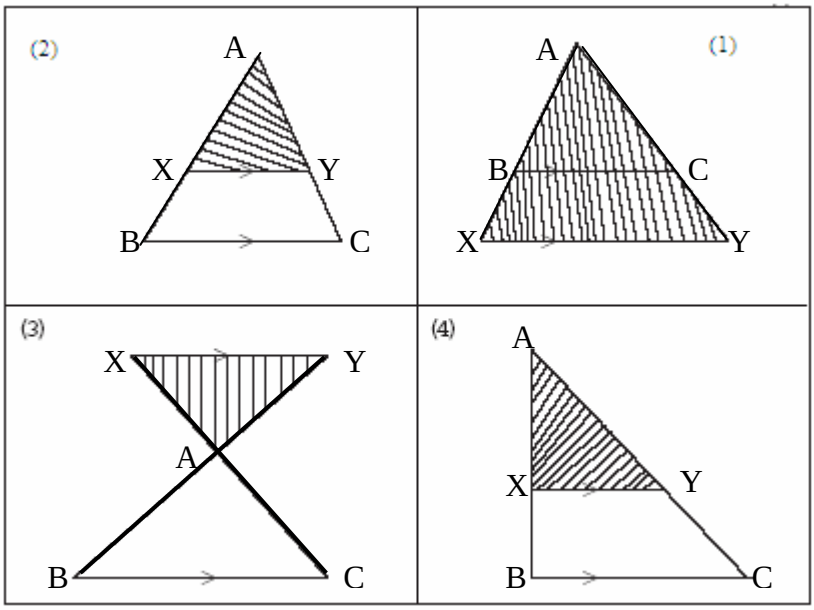
Annex 23.4.2

Instructions for group exploration

Let us identify the properties of equi-angular triangles

Part I

- Focus your attention on the figure obtained by your group from the following figures.



- Develop a relationship between the sizes of the angles of the triangle ABC and the shaded triangle AXY.
- Are these two triangles congruent or not?
- Discuss whether it is appropriate to call these two triangles equi-angular triangles.

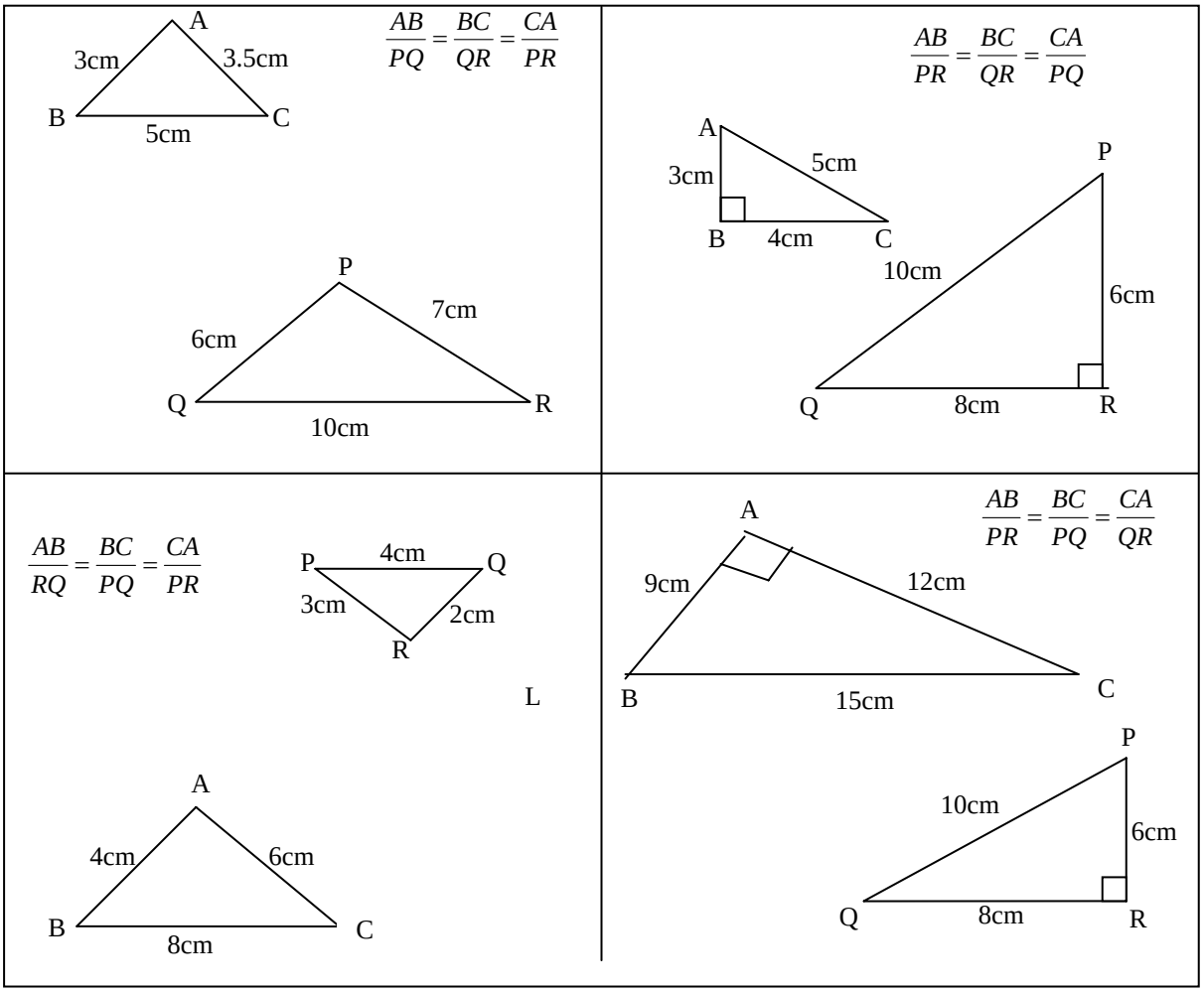
The corresponding sides of two equi-angular triangles are proportional

- Identify the corresponding sides of the triangle ABC and the shaded triangle AXY that you received.
- Examine whether the above mentioned theorem is true/false.
- Discuss whether the above theorem can be proved using the theorem that the straight line drawn parallel to one side of a triangle cuts the other two sides proportionally.
- Prepare to make a creative presentation of your findings at the plenary session.

Part II

If the three sides of a triangle are proportional to the three sides of another triangle, then the two triangles are equi-angular

- Focus your attention on the pair of triangles with corresponding sides proportional, obtained by your group from the following pairs of triangles.



- Examine whether the converse of the above theorem is true or false by either cutting the triangles and overlapping the angles or by some other appropriate method.
- Prepare to make a creative presentation of the group's findings at the plenary session.

13. Quadratic Equations

Competency 17 : Manipulates the methods of solving equations to fulfill the needs of day to day life.

Competency Level 17.1 : Investigates how quadratic equations may be used to solve problems that arise in daily life.

Activity 17.1 : Let us solve quadratic equations.

Time : 120 minutes.

Quality Inputs :
- An enlarged copy of the leaflet included in Annex 17.1.1.
- Two copies of the instruction leaflet on exploration included in Annex 17.1.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 17.1.1 :
- Present the leaflet on factors to the class and inquire from the students about finding the factors of a trinomial quadratic expression and about completing squares.
- Lead a discussion and highlight the following facts.
  - That the factors of a quadratic expression of the form $ax^2 + bx + c$ can be obtained if b is the sum of two factors of $a \times c$, which when multiplied together equals $a \times c$
  - That to complete the square of an expression of the form $x^2 + bx$, $\left(\frac{b}{2}\right)^2$ should be added to the expression

(20 minutes)

Step 17.1.2 :
- Divide the class into two small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 17.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.

- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the factors of the expression $x^2 + bx + c$ in the equation $x^2 + bx + c = 0$ can be found, if c can be written as a product of two factors, such that the sum of the two factors equals b
- That two simple equations are obtained by equating the two factors separately to 0
- That by solving them two values are obtained for x
- That any quadratic equation has two solutions

(20 minutes)

Step 17.1.4 **:**
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for presentations at the plenary session.

(20 minutes)

Step 17.1.5 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That 'completing the square' can be used to solve a quadratic equation of the form $ax^2 + bx + c = 0$
- That a formula in terms of a, b, c can be developed to solve a quadratic equation of the form $ax^2 + bx + c = 0$
- That any quadratic equation can be solved using the formula
- That the solutions to a quadratic equation obtained by completing the square and by using the formula are equal

(30 minutes)

Criteria for Assessment and Evaluation:
- Describes how 'completing the square' is used to solve quadratic equations.
- Accepts that a formula can be developed to solve quadratic equations easily.
- Solves quadratic equations using the formula.
- Logically investigates two equal instances.
- Approaches a specific goal by following instructions.

Annex 17.1.1

Factors of a Trinomial Quadratic Expression

(i) $x^2 + 8x + 15$

$= x^2 + 5x + 3x + 15$

$= x(x+5) + 3(x+5)$

$= (x+5)(x+3)$

(+15)

(+5) × (+3)

(ii) $x^2 + 2x - 24$

$= x^2 + 6x - 4x - 24$

$= x(x+6) - 4(x+6)$

$= (x+6)(x-4)$

(-24)

(+6) × (-4)

Completing the Square

(iii) $x^2 + 10x + ... = x^2 + 10x + 5^2 \quad \leftarrow \left(\frac{10}{2}\right)^2$

$= (x+5)^2$

(iv) $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

(v) If $a \times b = 0$ then $a = 0$ or $b = 0$

Annex 17.1.2

Instructions for group exploration

Let us solve quadratic equations

Part I

- Focus your attention on the case given to your group from the two cases A and B given below.

A	Develop an equation of the form $x^2 + bx + c = 0$ by selecting values for b and c such that b is the sum of two factors of c which when multiplied equals c
B	Develop an equation of the form $x^2 + bx - c = 0$ by selecting values for b and c such that b is the difference of two factors of c which when multiplied equals c

- Factorize the left hand side of the equation that you developed.
- Using these factors, write down two simple equations and find x by solving them.
- Discuss within the group why two values are obtained for x.
- Prepare to creatively present your findings to the class.

Part II

- Focus your attention on the case given to your group from the two cases A and B given below.

A	Obtain a quadratic equation by substituting $a = 1$ and two integers for b and c, in the equation $ax^2 + bx + c = 0$
B	Obtain a quadratic equation by substituting in the equation $ax^2 + bx + c = 0$, three integers for a, b and c such that b is a multiple of a

- If the coefficient of x^2 in the equation you obtained is not equal to 1, divide the equation so that the coefficient of x^2 equals 1.
- Take the constant term to the right hand side and make the left hand side into a perfect square.
- If a negative value is obtained on the right hand side, carryout the same process after substituting different values for a, b and c, so that the value on the right hand side is positive .
- Find the square roots of both sides and obtain the values of x.
- Examine the accuracy of the values obtained for x by substituting them back into the equation.
- Divide the equation $ax^2 + bx + c = 0$ by a, so that the coefficient of x^2 is 1.
- Develop a formula for x in terms of a, b and c by using the method of completing squares.
- Discuss about the values that you obtain for x by substituting the values of a, b and c from the first equation in the formula.
- Prepare to creatively present your findings to the class.

14. Graphs

Competency 20 : Easily communicates the mutual relationships that exist between two variables by exploring various methods.

Competency Level 20.1 : Solves problems using graphical methods.

Activity 20.1 : Let us find different methods of solving simultaneous equations.

Time : 90 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 20.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 20.1.1 :
- Present an equation such as $y = x + 3$ to the class and inquire from the students about finding the coordinates to draw the linear graph and about drawing the graph.
- Lead a discussion and highlight the following facts.
  - That the graph of an equation of the form $y = ax \pm b$, $a > 0$, makes an acute angle with the positive x- axis
  - That the graph of an equation of the form $y = ax \pm b$, $a < 0$, makes an obtuse angle with the positive x- axis
  - That to draw the graph, it is sufficient to substitute 3 values for x and find the corresponding 3 values of y

(30 minutes)

Step 20.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(25 minutes)

Step 20.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the solution of two simultaneous equations can be obtained by graphing the equations and finding the coordinates of the point of intersection of the two straight lines
- That the accuracy of the solution can be checked by substituting it into each of the equations
- That the simultaneous equations have no solutions when the graphs of the two equations are parallel

(35 minutes)

Criteria for Assessment and Evaluation:

- Expresses that the solution of a pair of simultaneous equations is obtained by the intersection point of the graphs of the two equations.
- Accepts that the solution of two equations consisting of two variables can be easily obtained by pictorial representation.
- Obtains the solution of a pair of simultaneous equations using graphs.
- Uses various methods to solve problems.
- Helps to develop relationships.

Annex 20.1.1

Instructions for group exploration

Let us find different methods of solving simultaneous equations

- Focus your attention on the pair of simultaneous equations obtained by your group from the following pairs of simultaneous equations.

Group	Pair of Simultaneous Equations
A	$y = 2x + 1$ $y = -\frac{1}{2}x + 5$
B	$y = 2x - 2$ $y = 2x + 4$
C	$y = 2x + 5$ $y = -x + 2$

- Solve the pair of simultaneous equations.
- Discuss and find a method of obtaining the solution of the simultaneous equations by using your knowledge of graphs and without solving the equations.
- If the pair of equations cannot be solved, discuss the reasons for it.
- Prepare for a creative presentation of your findings, showing the relationship between the solution obtained graphically and the one obtained by solving the equations.

14. Graphs of Quadratic Functions

Competency 20 : Easily communicates the mutual relationships that exist between two variables by exploring various methods.

Competency Level 20.2 : Analyses the characteristics of a quadratic function by its graph.

Activity 20.2 : Let us draw the graphs of quadratic functions; let us consider their characteristics.

Time : 90 minutes.

Quality Inputs :
- The Chart of Figures included in Annex 20.2.1
- Four copies of the instruction leaflet on exploration included in Annex 20.2.2
- Graph paper and logarithmic tables.
- Demy papers and pastels.

Learning – Teaching Process:

Step 20.2.1 :
- Present the chart of figures to the class.
- Lead a discussion and highlight the following facts.
  - That in functions of the form $y = ax^2 \pm b$, if $a > 0$, the minimum value and if $a < 0$, the maximum value is $\pm b$
  - That the graph of $y = ax^2 \pm b$ is obtained by the displacement of the graph of the function $y = ax^2$ by $\pm b$ units up/down the y –axis
  - That the graph of $y = a(x \pm b)^2$ is obtained by the displacement of the graph of $y = ax^2$ by $\pm b$ units to the right or the left along the x-axis

(20 minutes)

Step 20.2.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, graph paper, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(40 minutes)

Step 20.2.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the equation of the axis of symmetry of the graph of the function $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$
- That the values of x for which $y = 0$ in functions of the form $y = ax^2 + bx + c$ are the roots of the equation $ax^2 + bx + c = 0$
- That the axis of symmetry of functions of the form $y = (x \pm a)^2 \pm b$ is $x = \pm a$ while the minimum value is $\pm b$

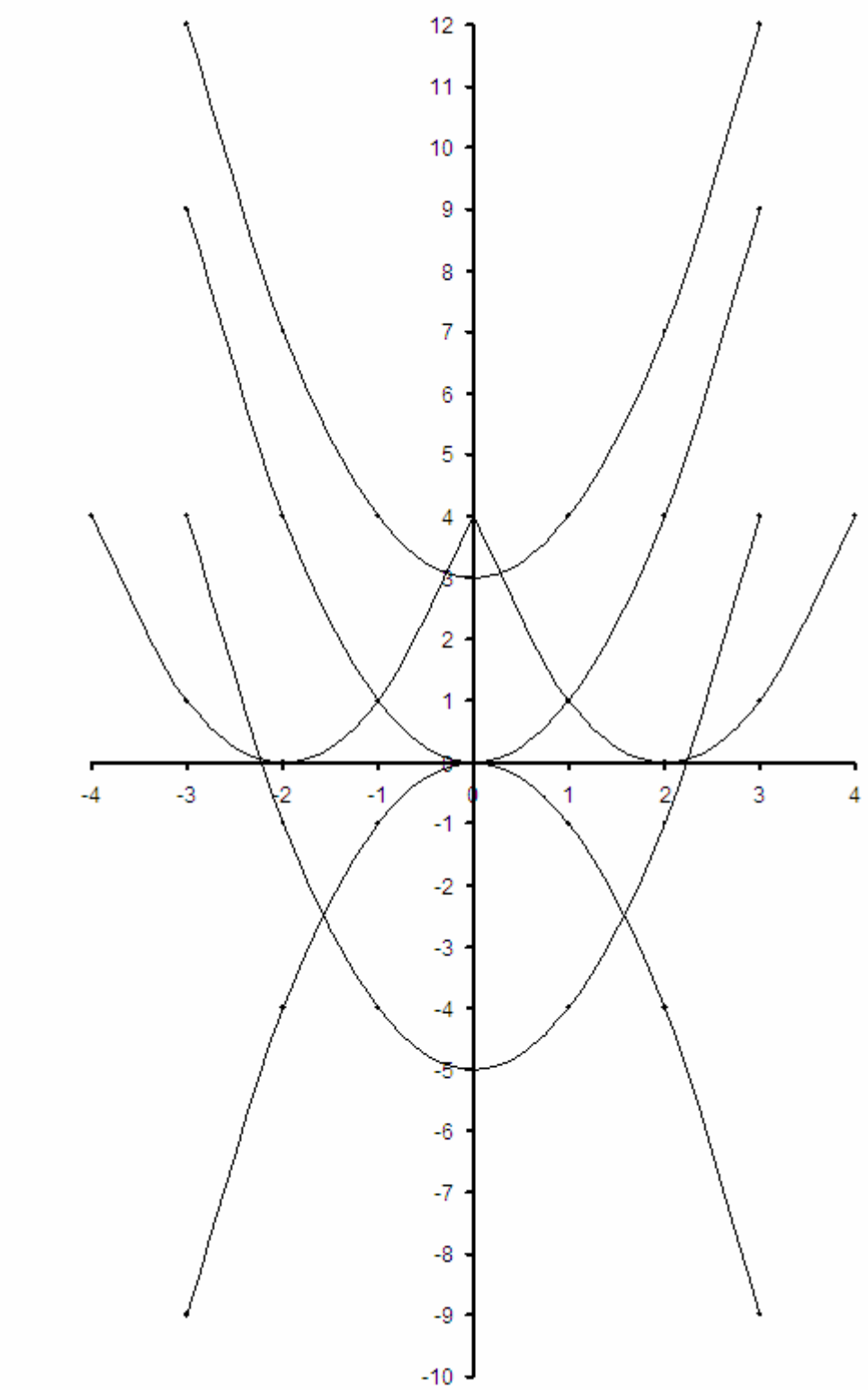
(30 minutes)

Criteria for Assessment and Evaluation:

- Describes the characteristics of quadratic functions.
- Accepts that it is possible to describe the behaviour of a quadratic function without drawing its graph.
- Solves quadratic equations using the graphs of quadratic functions.
- Succeeds in carryout out a task by working methodically step by step.
- Facilitates investigations using pictorial representations.

Annex 20.2.1

Chart of Figures



Annex 20.2.2

Instructions for group exploration

Let us draw the graphs of quadratic functions

- Focus your attention on the quadratic function obtained by your group from the following quadratic functions.
  - $y = x^2 - 2x - 3$
  - $y = (x-1)^2$
  - $y = (x-1)^2 - 2$
  - $y = 3 + 2x - x^2$
- Take the values of x to range from -2 to +4, prepare a table of values for y and draw the graph of the function.
- Solve by the completing the square, the quadratic equation that you get by substituting $y = 0$ into your function.
- What is the relationship between the values that you got for x by solving the equation and the values of x for which $y = 0$ in the graph?
- Discuss and write a method to obtain without drawing the graph, the maximum/minimum value and the equation of the axis of symmetry of a quadratic function.
- Creatively prepare for a presentation of your findings at the plenary session.

15. Representation and Interpretation of Data I

Competency 28 : Facilitates daily work by investigating the various methods of representing data.

Competency Level 28.1 : Represents data graphically.

Activity 28.1 : Let us represent information by a histogram.

Time : 90 minutes.

Quality Inputs :
- The Chart included in Annex 28.1.1
- Four copies of the instruction leaflet on exploration included in Annex 28.1.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 28.1.1 :
- Present the chart to the class and inquire from the students about histograms and the theoretical limits of the class intervals.
- Lead a discussion and highlight the following facts.

- That the information given in a table with class intervals can be represented in a histogram
- That the areas of the bars of a histogram are proportional to the frequencies of the relevant class intervals
- That when the class intervals are equal, the height of the bars are equal to the frequencies
- That when the initial part of the horizontal axis is not required to draw the histogram, this is represented by a zig zag line segment
- That quantities such as the number of children, number of fruits, number of books etc, which are whole numbers, are discrete quantities
- That quantities such as length, weight, times etc which are not necessarily whole numbers are continuous quantities
- That when drawing histograms, discrete data has to be converted into continuous form
- That the lower boundary of a class interval is the mid-point of the lower limit of the interval and upper limit of the previous class interval

(15 minutes)

Step 28.1.2 : • Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 28.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That a frequency distribution given in a table with unequal class intervals can be represented by a histogram
- That when deciding on the height of a bar of the histogram, the size of the smallest class interval is taken into consideration
- That the height of a bar reduces depending on how many times larger the relevant class interval is with respect to the size of the smallest class interval in the table
- That if the size of a class interval is n times the size of the smallest class interval, then the height of the relevant histogram bar is $\frac{f}{n}$ (frequency $\div n$)

(20 minutes)

Step 28.1.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 28.1.5 : • Provide each small group with the opportunity to present the findings of the group.

- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That histograms can also be drawn for frequency tables of discrete quantities • That when drawing the histogram, the class boundaries are used to remove the gaps • That by applying the class boundaries, the class intervals of frequency distributions of discrete quantities can be converted to the form of class intervals for continuous quantities

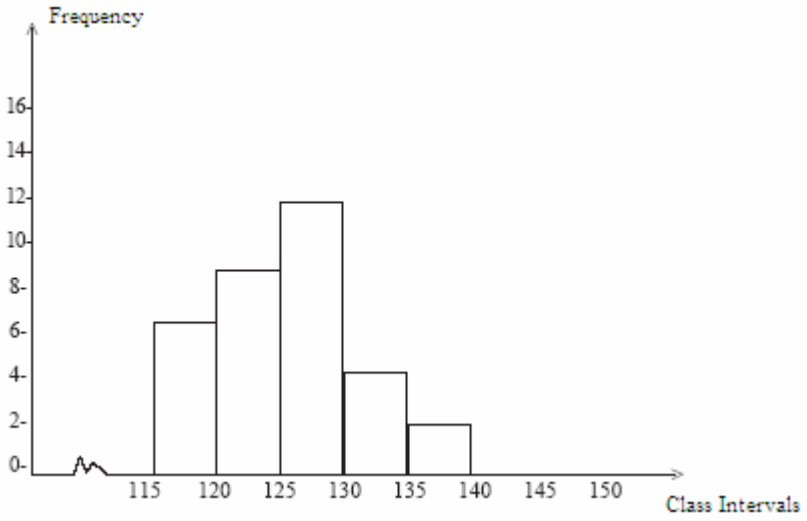
(15 minutes)

Criteria for Assessment and Evaluation:

- Expresses various ideas using the information obtained from frequency tables with unequal class intervals.
- Accepts that information given in tabular form can be easily communicated by representing it graphically also.
- Draws the histogram for a frequency table with unequal class intervals.
- Manipulates with awareness for one's own needs, the data that has been represented by various methods.
- Comes to common decisions by considering other opinions too.

Annex 28.1.1

Graph



Annex 28.1.2

Instructions for group exploration

Let us represent information by a histogram

Part I

- Focus your attention on the case given to your group from the cases given below.

(a) The following table has been prepared using the lengths of the pieces of cloth of a stock remaining in a store.

Class Interval (Length of a piece in cm)	70 - 90	90 - 110	110 - 130	130 - 150	150 - 190	190 - 210
Frequency (Number of Pieces)	5	6	10	8	8	2

(Here, 70 – 90 means more than 70 and less than or equal to 90, etc)

(b) The following table has been prepared by using the times taken by a group of students to read a page of the English textbook.

Class Interval (Time in minutes)	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 16
Frequency (Number of students)	3	5	8	6	4	4

(Here, 2 – 4 means more than 2 and less than or equal to 4, etc)

(c) The following table has been prepared using the milk yield from cows on a certain day at a certain farm.

Class Interval (Milk in litres)	0 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8
Frequency (Number of cows)	15	10	20	18	12	5

(Here, 0 – 3 means more than 0 and less than or equal to 3, etc)

(c) The following table has been prepared using the weights of a stock of papaws.

Class Interval (Weight of a fruit in g)	500 - 600	600 - 700	700 - 800	800 - 900	900 - 1100	1100 - 1300
Frequency (Number of fruits)	5	8	10	15	14	12

(Here, 500 – 600 means more than 500 and less than or equal to 600, etc)

- Draw a histogram using the given frequency table, paying special attention to the heights of the bars when the class intervals are unequal.
- Discuss the special methods used to draw the bars of the histogram such that the areas are proportional to the frequencies.
- Prepare to creatively present your findings to the class.

Part II

- Focus your attention on the case given to your group from the cases given below.

(a) The following table has been prepared using the number of trips made in a day by vehicles in a taxi service.

Class Interval (Number of trips)	1 - 3	4 - 6	7 – 9	10 – 12	13 - 15
Frequency (Number of Vehicles)	5	6	8	4	1

(b) The following table has been prepared using the number of lottery tickets sold by members of a certain organization to collect money for a common cause.

Class Interval (Number of Tickets)	21 - 40	41 - 60	61 - 80	81 - 100	101 - 120
Frequency (Number of members)	15	17	20	28	12

(c) The following table has been prepared using the scores a cricketer obtained in 50 matches.

Class Interval (Scores)	1 - 20	21 - 30	31 - 40	41 - 50	51 - 60
Frequency (Number of matches)	8	11	15	12	4

(c) The following table has been prepared using the number of students in each class of a certain school.

Class Interval (Number of students)	15 - 19	20 - 24	25 - 29	30 - 34	35 - 43
Frequency (Number of classes)	3	7	6	5	4

- Organize the class intervals as for continuous data using class boundaries, and draw a histogram for the given frequency table, paying special attention to the boundaries.
- Discuss the importance of applying class boundaries when drawing histograms for discrete data.
- Prepare to present your findings at the plenary session.

15. Representation and Interpretation of Data II

Competency 28 : Facilitates daily work by investigating the various methods of representing data.

Competency Level 28.2 : Develops relationships between the various graphs that represent data.

Activity 28.2 : Let us draw frequency polygons by using the given information.

Time : 60 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 28.2.1
- Demy papers and pastels.

Learning – Teaching Process:

Step 28.2.1 :
- Inquire from the students about frequency tables, histograms and the mid-values of class intervals and polygons.
- Lead a discussion and highlight the following facts.

  - That the information given in a frequency table can be represented in a histogram
  - That histograms can be drawn for frequency distributions of continuous as well as discrete variables
  - That the mid point of a class interval maybe obtained by taking the sum of the two class limits and dividing it by 2
  - That a plane figure bounded by straight line segments is defined as a polygon

(20 minutes)

Step 28.2.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 28.2.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.

- Engage in a review so that the following facts are highlighted.
  - That the area of the frequency polygon drawn using data from a frequency table is the same as the area of the histogram drawn for the same frequency table (Figure 1)
  - That the frequency polygon can be drawn by joining the mid-points of the upper end of the frequency bars and the mid-points of the class intervals on the two sides on the axis with the observation values
  - That the frequency polygon can also be drawn by considering the mid-points of the class intervals (Figure II)

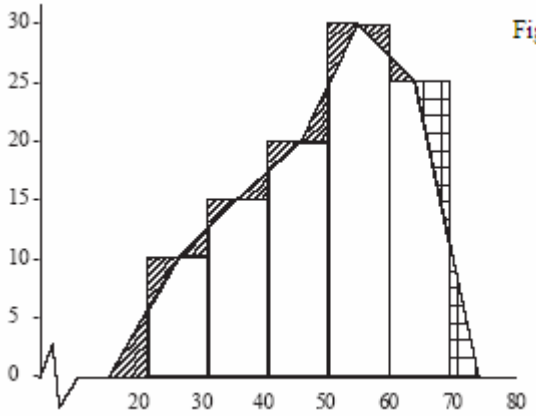


Figure I

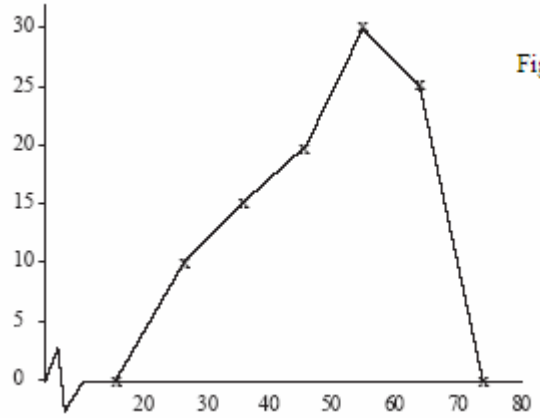


Figure II

(20 minutes)

Criteria for Assessment and Evaluation:

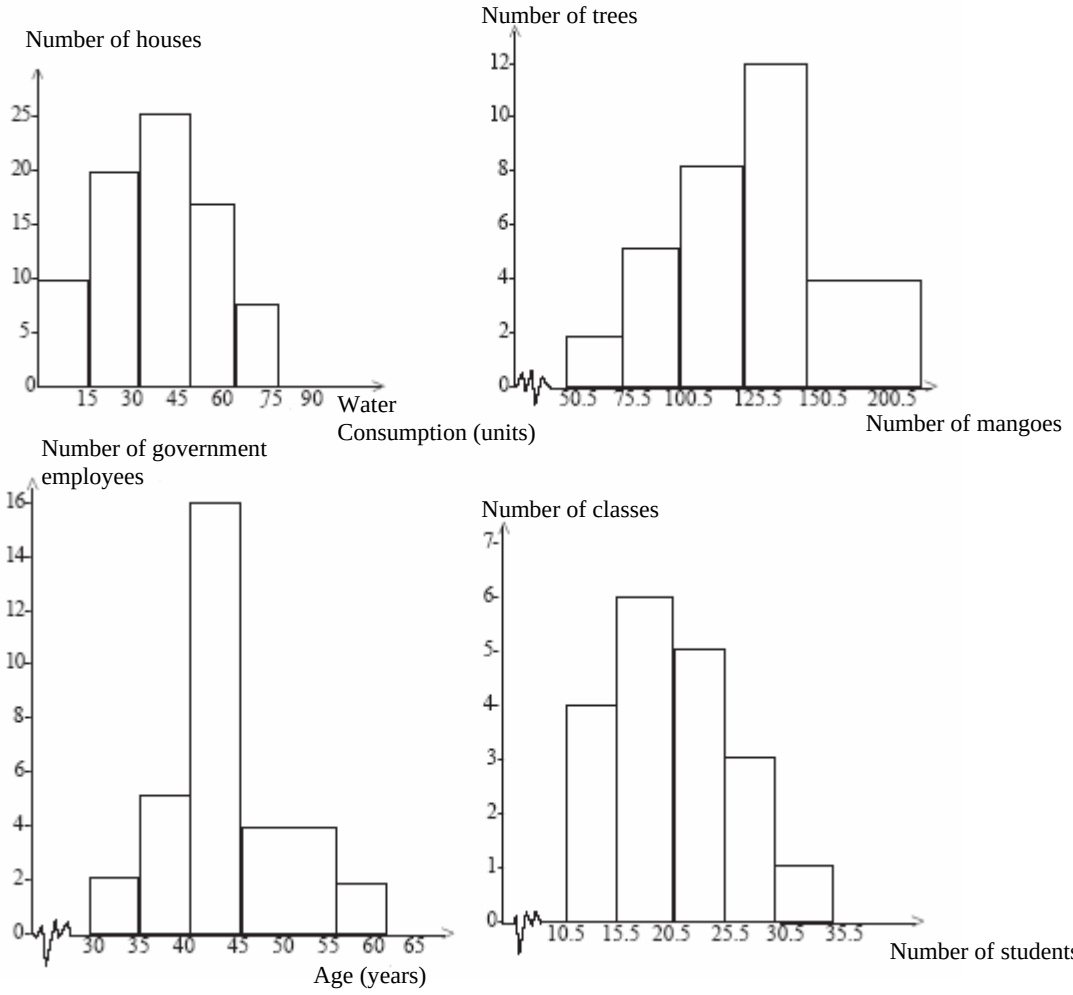
- Explains how the frequency polygon is made into a closed figure such that its area is equal to the area of the histogram.
- Accepts that by thinking logically, methods of easily representing data can be discovered.
- Draws the frequency polygon using the given histogram.
- Uses figures to represent complex things in a simpler manner.
- Comes to the same conclusion by different methods.

Annex 28.2.1

Instructions for group exploration

Let us draw frequency polygons by using the given information

- Focus your attention on the figure given to your group from the figures given below.



- Copy the given histogram and obtain a polygon that has the same area as the histogram by joining the mid-points of the upper ends of the frequency bars.
- Describe how the mid-points of the upper ends of the frequency bars of the first and last class intervals should be joined to the axis with the class intervals for the area of the frequency polygon formed in this manner to be the same as the area of the histogram.
- Determine whether there are other methods by which the above frequency polygon can be constructed.
- Prepare for a presentation at the plenary session.

15. Representation and Interpretation of Data III

Competency 28 : Facilitates daily work by investigating the various methods of representing data.

Competency Level 28.3 : Represents the relationships between the cumulative frequency curve and the boundaries of a group of data.

Activity 28.3 : Let us draw a cumulative frequency curve.

Time : 120 minutes.

Quality Inputs :
- The Chart of Figures included in Annex 28.3.1.
- Two copies of the instruction leaflet on exploration included in Annex 28.3.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 28.3.1 :
- Display the chart of figures in the classroom.
- Lead a discussion and highlight the following facts by interpreting the given data.
  - That the total frequency can be found using the table
  - That the number who got less than a certain score can be found by taking the sum of the frequencies that are below that score
  - That the number who got more than a certain score can be found by taking the sum of the frequencies that are above that score

(20 minutes)

Step 28.3.2 :
- Divide the class into two small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (20 minutes)

Step 28.3.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the column of cumulative frequencies is obtained by adding the numbers in the column of frequencies in the table from top to bottom or from bottom to top
- That the graph drawn using the cumulative frequencies is called the cumulative frequency curve
- That the median can be found by reading from the graph, the value corresponding to the mid-value of the number of data points
- That the mid-values of the upper and lower parts of the graph formed by the median, are defined as quartiles
- That the lower quartile is denoted by Q_1, the median by Q_2 and the upper quartile by Q_3
- That the inter-quartile range is given by $(Q_3 - Q_2)$

(30 minutes)

Step 28.3.4 **:**

- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (20 minutes)

Step 28.3.5 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That cumulative frequency curves can also be drawn for tables with grouped data
- That for a table of grouped data written in ascending order, the number of values that is less than the upper limit of each interval is given by the corresponding value of the cumulative frequency curve
- That for a table of grouped data written in descending order, the number of values that is greater than the upper limit of each interval is given by the corresponding value of the cumulative frequency curve
- That the median and quartiles can be obtained by using the cumulative frequency curve drawn to represent grouped data also

- That the median can be obtained by the intersection of the greater cumulative curve and the lesser cumulative curve of a group of data

(30 minutes)

Criteria for Assessment and Evaluation:

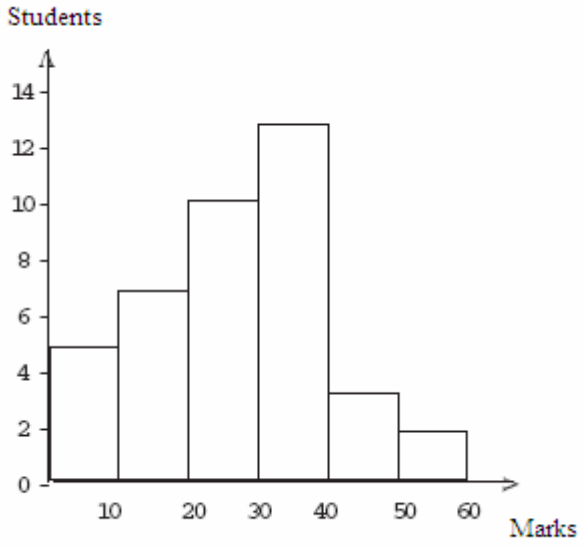
- Expresses the cumulative frequencies when a frequency table is given.
- Accepts that representative values such as the median can easily be obtained by representing data by a cumulative frequency curve.
- Draws the cumulative frequency curve by first preparing the relevant pair of axes.
- Approaches the final goal by carrying out instructions.
- Makes decisions easily by understanding the abstract through the concrete.

Annex 28.3.1

Figure

A table with the scores that a certain class of grade 11 students obtained for the Mathematics I question paper and the histogram corresponding to it are given below.

Range of Scores	Frequency (Number of Students)
0-10	05
10-20	07
20-30	10
30-40	13
40-50	03
50-60	02



Annex 28.3.2

Instructions for group exploration

Let us draw a cumulative frequency curve

Part I

- The manner in which two groups of children obtained marks out of five for an exercise are given below

Group 1		
Marks	Frequency	
0	1	
1	4	
2	6	
3	8	
4	5	
5	3	

Group 2		
Marks	Frequency	
5	1	
4	3	
3	7	
2	7	
1	6	
0	3	

- Focus your attention on the table given to your group from the tables given above and study it carefully.
- Observe whether the information on the marks has been recorded in ascending order or in descending order.
- Find out how many students got the mark given in the first row of the first column of the table.
- Record it first in the third column.
- Students who got less than or equal to/greater than or equal to which mark do we get, when we find the total number of students who got marks related to the first two marks in the first column?
- In the same manner, for each row, find the number of students who got less than or equal to or more than or equal to the mark given in that row and table it in the 3^{rd} column. Propose a name for that column.
- Draw a graph with the values in the 'Marks' column on the x – axis and the values in the 3^{rd} column on the y-axis.
- Discuss using the graph how the mark of the student who is in the middle can be found.
- Find the marks relevant to the mid-values of the two parts that the graph is separated into by the mid-value of the graph.
- Find the difference between these two marks.
- Propose suitable names for these mid-values.
- Prepare to present your findings at the plenary session.

Part II

- Two table with the scores that a certain class of grade 11 students obtained for the Mathematics I question paper are given below.

Group 1	
Marks	Frequency
00 - 10	05
10 - 20	08
20 - 30	10
30 - 40	12
40 - 50	03
50 - 60	02

Group 2	
Marks	Frequency
60 - 50	02
50 - 40	03
40 - 30	12
30 - 20	10
20 - 10	08
10 - 0	05

- Focus your attention on the table given to your group from the tables given above and study it carefully.
- Obtain a cumulative frequency column.
- Discuss above/below what mark the numbers in the rows of the cumulative frequency column represent.
- Select the limits of the class intervals relevant to draw the graph, and draw the graph with these limits as the x –values and the cumulative frequencies as the y – values.
- Obtain the values in the cumulative frequency column of the other group and draw the corresponding cumulative frequency curve on the same paper that you drew the first graph.
- Discuss about the mark obtained by the intersection point of the two curves.
- Prepare for a presentation at the plenary session.

16. Geometric Progression

Competency 2 : Makes decisions for future requirements by investigating the various relationships in number patterns

Competency Level 2.1 : Investigates the various behavioral patterns of progressions using sequences of numbers.

Activity 2.1 : Let us analyze geometric progressions.

Time : 140 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 2.1.1.
- Demy papers and pastels

Learning – Teaching Process:

Step 2.1.1 :
- Present the following sequences of numbers to the class
  2, 5, 8, 11, …..
  10, 8, 6, 4, …..
  $a, a + d, a + 2d, a + 3d, \ldots..$
- Inquire from the students about the next term and about the sequence.
- Lead a discussion and highlight the following facts.

- That the above sequences of numbers have a common difference (by subtracting from a term its previous term)
- That sequences of numbers with a common difference are arithmetic progressions
- That the n^{th} term of an arithmetic progression with initial term a and common difference d is $T_n = a + (n-1)d$
- That the sum of the n terms of an arithmetic progression with initial term a and last term l is $S_n = \frac{n}{2}(a + l)$
- That there is a relationship between the terms of a progression

(20 minutes)

Step 2.1.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.

- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 2.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the common ratio of a geometric progression is obtained by dividing a term by the previous term
- That progressions with a common ratio are called geometric progressions
- That when the initial term is a and the common ratio is r, the n^{th} term is given by $T_n = ar^{n-1}$
- That if a , b, c are three consecutive terms of a geometric progression,
  the geometric mean $= \pm\sqrt{ac}$,
  $b = \pm\sqrt{ac}$

(30 minutes)

Step 2.1.4 :
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (30 minutes)

Step 2.1.5 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That when the initial term of a geometric progression is a, the common ratio is r, and the number of terms is n, the sum of the n terms is $S_n = \dfrac{a(r^n - 1)}{r - 1}$
- That when finding the sum of a large number of terms of a geometric progression, it is easier to use the formula

(30 minutes)

Criteria for Assessment and Evaluation:

- Names the progression by considering the relationship between consecutive terms of a sequence of numbers.
- Accepts that it is easy to find the sum of a large number of terms of a sequence of numbers by using a formula.
- Finds any term and the sum of any number of terms using formulae.
- Solves problems easily by understanding the pattern.
- Simplifies complex processes by applying easy methods.

Annex 2.1.1

Instructions for group exploration

Let us analyze geometric progressions

Part I

2, 6, 18, 54, ….

- Discuss within the group how the terms of the above sequence of numbers have been obtained from the previous term and extend the sequence by three steps.
- Elicit the relationship between two consecutive terms.
- Compare the relationship that you obtained with the properties of an arithmetic progression.

Sequences of numbers which have a common ratio between consecutive terms are defined as geometric progressions

- Focus your attention on the information that your group receives from the information given below.
  - The initial term is 2 and the common ratio is 3
  - The initial term is 2 and the common ratio is -3
  - The initial term is 2 and the common ratio is $\frac{1}{2}$
  - The initial term is 2 and the common ratio is 0.5

- Extend the progression up to 6 terms using the information that your group received.
- Be attentive to the terms of the progression that you developed.
- Propose a method to find the 10^{th} term of the progression.
- If the first term is a and the common ratio is r, use the relationship you obtained above to develop a formula for the n^{th} term.
- Find the value of a term further down the progression and confirm the accuracy of your formula.
- If three consecutive terms of the progression are a, b and c, write down the relationship between a and b, and between b and c, and use these to obtain an expression for b.
- Propose a suitable name for b.
- Prepare to make a creative presentation of your findings at the plenary session.

Part II

- Focus your attention on the geometric progression that your group receives from the geometric progressions given below.

  i. $a = 3 \quad r = 2$
  ii. $a = 3 \quad r = -2$
  iii. $a = 16 \; r = \frac{1}{2}$
  iv. $a = 16 \; r = -\frac{1}{2}$

- Study from your textbook how the formula $S_n = a\frac{(r^n - 1)}{(r - 1)}$ has been developed for the sum of n terms of a geometric progression with initial term a and common ratio r.
- Substitute the information regarding the geometric progression received by your group into the formula and obtain the sum of the first 6 terms.
- Now, write down the first 6 terms of the geometric progression, and find their sum.
- Compare and discuss about the values that you obtained by using the formula and by adding the first 6 terms.
- Give an example from day to day life of an instance in which the values increase or decrease according to a geometric progression.
- Discuss why it is necessary to have a formula to find the sum of the terms of a geometric progression.
- Prepare for a creative presentation at the plenary session.

17. Pythagoras' Theorem

Competency 23 : Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency Level 23.5 : Examines the relationship between the sides of a right-angled triangle.

Activity 23.5 : Let us examine Pythagoras' relationship.

Time : 135 minutes.

Quality Inputs :
- Three copies of the Figure included in Annex 23.5.1.
- Three copies of the instruction leaflet on exploration included in Annex 23.5.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 23.5.1 :
- Inquire from the students how the length of the third side of a right-angled triangle is found when the lengths of two sides are given.
- Lead a discussion and highlight the following facts.

- That the side opposite to the right angle, which is the longest side of a right-angled triangle is known as the hypotenuse
- That if the length of the hypotenuse is a and the other two sides are of lengths b and c respectively, then $a^2 = b^2 + c^2$
- That when the lengths of two sides of a right-angled triangle are given, the length of the other side can be calculated
- That (3, 4, 5), (5, 12, 13) etc are know as Pythagorean triples

(15 minutes)

Step 23.5.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, figures, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 23.5.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That the area of the square drawn on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares drawn on the other two sides
  - That Pythagoras' theorem can be verified by calculating the areas of the squares drawn on the sides of a right-angled triangle
  - That Pythagoras' theorem can also be verified by drawing semi-circles, equilateral triangles and quarter circles on the sides of a right-angled triangle and calculating their areas

(30 minutes)

Step 23.5.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(35 minutes)

Step 23.5.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That a right-angled can be accurately obtained using Pythagoras' theorem
  - That Pythagoras' theorem is applied in daily activities
  - That Pythagoras' theorem can be proved by various methods

(25 minutes)

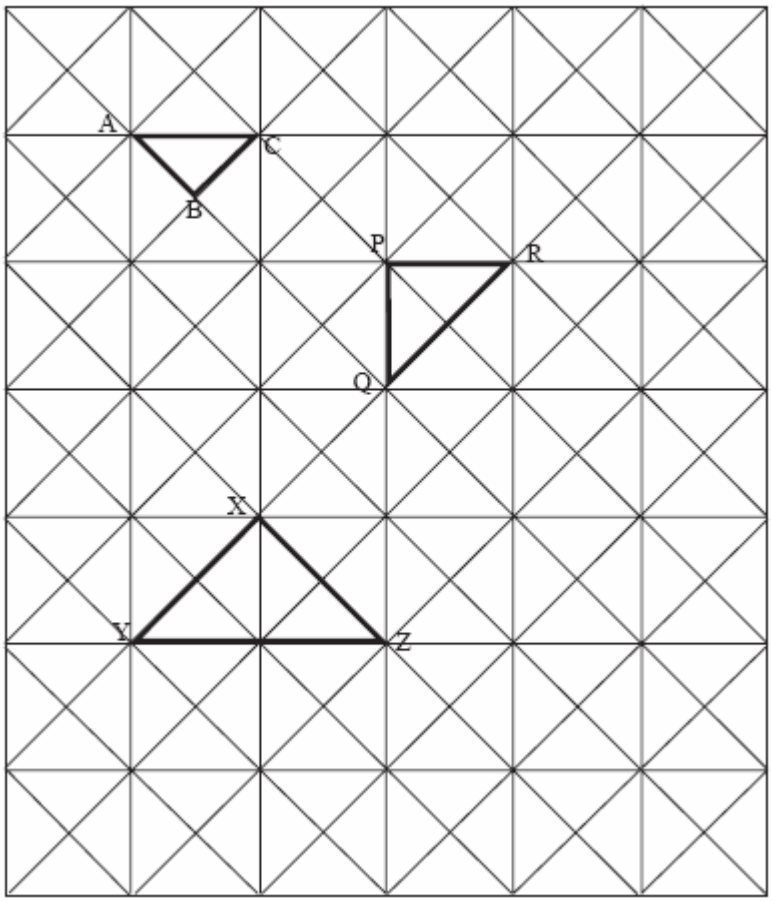
Criteria for Assessment and Evaluation:

- Accurately describes Pythagoras' relationship.
- Accepts that Pythagoras' theorem can be applied as an easy method of examining perpendicularity.
- Uses Pythagoras' relationship to construct a right-angle.
- Makes decisions logically.
- Approaches the solution from various sides.

Annex 23.5.1

Figure

The triangles ABC, PQR and XYZ that are given here are right-angled isosceles triangles

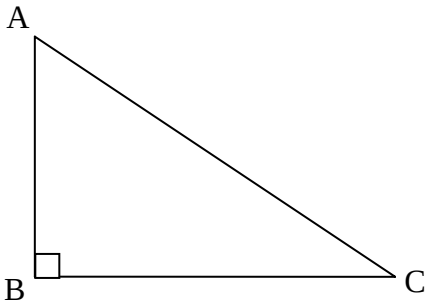


Annex 23.5.2

Instructions for group exploration

Let us examine Pythagoras' Relationship

Pythagoras' Theorem:
'In a right-angled triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides'



$AC^2 = AB^2 + BC^2$

Part I

- Focus your attention on the section that your group receives from the plane figures given below.

Section	Case I	Case II
(1)	The triangle ABC	Semi-circle
(2)	The triangle PQR	Equilateral Triangle
(3)	The triangle XYZ	Quarter circle

- Carefully examine the figures that you have received and determine whether Pythagoras' relationship is true for the triangle relevant to you from case I, by using the areas of the squares drawn on the sides of the triangle.
- Examine whether Pythagoras' theorem is true by constructing the plane figure relevant to your group from case II on the sides of a right-angled triangle with sides of length 6cm, 8cm and 10cm.
- Creatively prepare for a presentation at the plenary session.

Part II

- Focus your attention on the case that your group received from the cases given below.
- The triangle with AB = BC and $\hat{B}$ a right angle
- The triangle with AB > BC and $\hat{B}$ a right angle
- The triangle with AB < BC and $\hat{B}$ a right angle

- Draw BD the perpendicular from B to the hypotenuse AC.
- By considering the equi-angularity of the triangles ABD, BDC and ABC, prove Pythagoras' theorem for the triangle ABC.
- Discuss about the theorems you used in this proof and the reasons for using them.
- Discuss how Pythagoras' theorem can be used to draw a right angle without measuring angles.
- Prepare to make a creative presentation of the facts you found out at the plenary session.

18. Trigonometry

Competency 13 : Uses scale drawings in practical situations by exploring various methods.

Competency Level 13.1 : Manipulates trigonometric relationships for daily requirements.

Activity 13.1 : Let us solve problems using trigonometric tables.

Time : 130 minutes.

Quality Inputs :
- An enlarged copy of the poster in Annex 13.1.1.
- Three copies of the instruction leaflet on exploration included in Annex 13.1.2.
- Several copies of trigonometric tables.
- Demy papers and pastels.

Learning – Teaching Process:

Step 13.1.1 :
- Display the poster in the class and inquire from the students about solving problems by finding the trigonometric ratios of angles other than 30^o, 45^o and 60^o.
- Lead a discussion and highlight the following facts.

• That sin, cos and tan ratios can be defined using a right-angled triangle • That when the value of the angle increases, its sin and tan values also increase while the cos value decreases • That when considering trigonometric ratios of angles $\sin\theta = \cos(90^o - \theta)$ • That in the same way that the logarithm tables are used, the trigonometric value of an angle can be obtained from a trigonometric ratios table

(15 minutes)

Step 13.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, trigonometric tables, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(20 minutes)

Step 13.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That when obtaining the trigonometric ratios from the trigonometric tables
    - Since the sin and tan values increase with the increase in the value of the angle, when finding sin and tan ratios, the mean difference should be added
  - That since the cosine value decreases with the increase in the value of the angle, when finding the cosine value, the mean difference should be subtracted
  - That the relevant trigonometric ratio tables should be used to facilitate problem solving

(35 minutes)

Step 13.1.4 : • Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 13.1.5 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That to solve a problem, a figure with the data accurately marked should be drawn
  - That the correct trigonometric ratio table should be selected
  - That after the trigonometric ratios are obtained, if necessary, logarithmic tables should be used to carry out simplifications
  - That trigonometric ratios can be used to easily solve problems involving angles of elevation, angles of depression and bearings

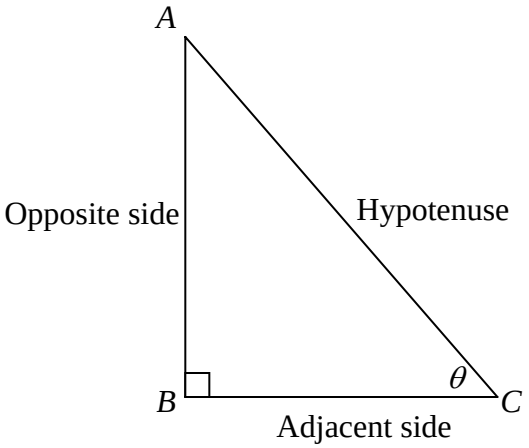
(40 minutes)

Criteria for Assessment and Evaluation:

- Expresses the trigonometric ratio of an angle using the trigonometric tables.
- Accepts the importance of including data accurately in a figure.
- Solves problems by using trigonometric ratios.
- Engages in the right process by methodically following steps.
- Facilitates the calculation of measurements by using pictorial representations.

Annex 13.1.1

Poster



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\tan\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{AB}{BC}$$

Angle	sin	cos	tan
0^o	0	1	0
30^o	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} = 0.8661$	$\frac{1}{\sqrt{3}} = 0.6773$
45^o	$\frac{1}{\sqrt{2}} = 0.7071$	$\frac{1}{\sqrt{2}} = 0.7071$	1
60^o	$\frac{\sqrt{3}}{2} = 0.8661$	$\frac{1}{2} = 0.5$	$\sqrt{3} = 1.7321$
90^o	1	0	-

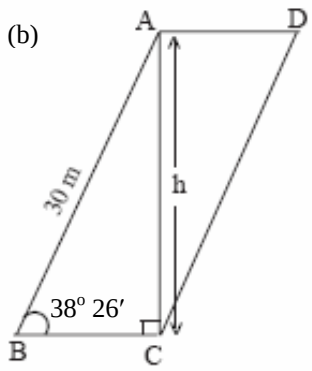
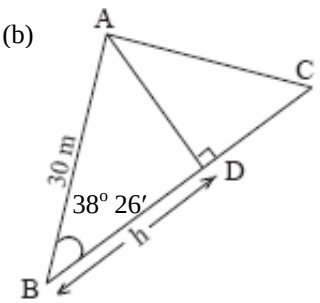
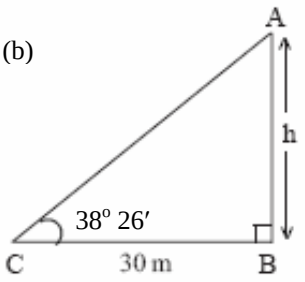
Annex 13.1.2

Instructions for group exploration

Let us solve problems using trigonometric tables

Part I

- Focus your attention on the figure obtained by your group from the figures given below.

A	B	C
(a) tan 45^o 28′	(a) sin 45^o 28′	(a) cos 45^o 28′
(b) 	(b) 	(b) 

- Study the trigonometric tables carefully and using the information given in the textbook discuss about the increase/decrease in values in the sine and cosine tables.
- Find the relationship with the mean difference when finding the trigonometric ratio of an angle.
- Select the appropriate trigonometric table to find the trigonometric value that your group received.
- Use the selected trigonometric table to find the trigonometric ratio given in (a).
- Write a relationship to find the value of *h* in figure (b), by using the relevant trigonometric ratio.
- Calculate the value of *h* using the trigonometric table relevant to your relationship.
- Prepare to make a creative presentation at the plenary session.

Part II

- Focus your attention on the event your group receives from the events given below.

A	B	C
• The angle of depression to a still boat which is seen by a person seated on a bund on the seashore, 35m from the boat is 28^o. Calculate the height of the bund.	• A child who starts from a point O, arrives at a point A which has bearings 040^o after traveling 50m. Determine how far north and east A lies from O.	• A child who is seated at a point A, holds a 40m long taut kite string in his hand. If he sees the kite at an angle of elevation of 30^o, calculate the vertical height of the kite from the ground.

- Draw a figure with the data stated in the event included in it.
- Solve the problem using trigonometric tables.
- Prepare to creatively present how you solved the problem at the plenary session.

19. Matrices

Competency 20 : Easily communicates the mutual relationships that exist between two variables by exploring various methods.

Competency Level 20.3 : Communicates the mutual relationships that exist between two variables using matrices.

Activity 20.3 : Let us communicate information easily.

Time : 75 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 20.3.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 20.3.1 :
- Inquire from the students about a concise way of representing the following information: Nimal buys 3 mangoes, 2 apples and 1 orange, while Kamal buys 2 mangoes, 3 apples and 4 oranges from a certain fruit stall.
- Lead a discussion and highlight the following facts.
  - That the information can be represented in a table with rows and columns
  - That there is some inconvenience in preparing the table when representing such information in tabular form
  - That there are other methods of representing data which facilitates communication

(15 minutes)

Step 20.3.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 20.2.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That information can be represented in a matrix such as $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ to facilitate communication
- That matrices are considered to be a means that facilitates the communication of information
- That the order of a matrix can be written as $x \times y$
- That x represents the number of rows and y the number of columns in the matrix
- That the order of a matrix is written at the bottom of the right hand side of the matrix as in $\begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \end{pmatrix}_{2\times 3}$
- That addition and subtraction of matrices is done by performing the operation separately on corresponding terms
- That repeated addition of a matrix is the same as multiplying the matrix by a whole number
- That when multiplying a matrix by a whole number, each term in the matrix should be multiplied by the whole number

(30 minutes)

Criteria for Assessment and Evaluation:

- Describes the conditions under which two matrices can be added and subtracted.
- Accepts that in comparison to preparing tables, representing information by matrices facilitates communication.
- Adds and subtracts two matrices of the same order.
- Selects appropriate methods to synthesize.
- Easily communicates information according to the need.

Annex 20.3.1

Instructions for group exploration

Let us communicate information easily

- Focus your attention on the set of numbers that your group receives from the following sets of numbers which are related to variables.

	$x \times y$
• $(1 \;\; 3)$	1×2
• $\begin{pmatrix} 5 & 7 \\ 6 & 4 \end{pmatrix}$	2×2
• $\begin{pmatrix} 2 & 3 & 1 \\ 5 & 1 & 2 \end{pmatrix}$	2×3
• $\begin{pmatrix} 2 & 9 & 1 \\ 7 & 5 & 3 \\ 4 & 2 & 6 \end{pmatrix}$	3×3

- Obtain ideas about the relationship between the number of rows and columns of the set of numbers and x and y.
- Taking x as children and y as fruits develop an example to explain the relationship.
- Identify the conditions that should be satisfied to add and subtract two sets of numbers of this form, by using your textbook.
- Accordingly, select two other sets of numbers that satisfy these conditions and engage in performing the two operations by identifying corresponding terms.
- Write the same set of numbers twice, add them and propose another method by which the solution can be easily obtained.
- Propose a name to define sets of numbers such as these.
- Prepare for a creative presentation of your group's findings at the plenary session.

20. Inequalities

Competency 18 : Analyzes the relationships between various quantities related to real-life problems.

Competency Level 18.1 : Solves problems involving the relationships between two quantities.

Activity 18.1 : Let us solve inequalities.

Time : 75 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 18.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 18.1.1 :
- Write an inequality of the form $ax \pm b > c$, and inquire from the students about the steps that should be followed in solving it.
- Lead a discussion and highlight the following facts.

• That an inequality remains unchanged when the same positive number or negative number is added to the two sides of the inequality • That an inequality remains the same when both sides of the inequality are multiplied or divided by the same positive number • That an inequality changes when both sides of the inequality are multiplied or divided by the same negative number

(15 minutes)

Step 18.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 18.1.3 : • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That when the relationship between the sizes of two algebraic expressions that are expressed in terms of the same quantity are known, they can be related with an inequality sign • That by solving inequalities, a solution is obtained for the unknown • That the solution can be represented on the number line • That a solution that satisfies several condition can be obtained by solving inequalities

(30 minutes)

Criteria for Assessment and Evaluation:

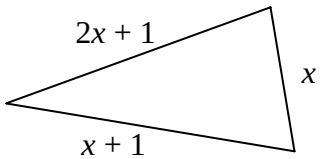
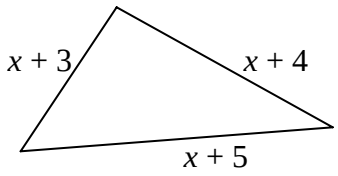
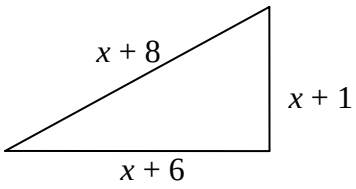
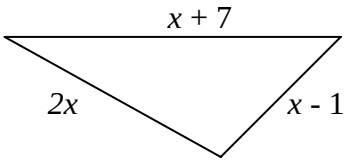
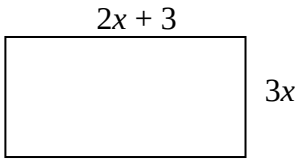
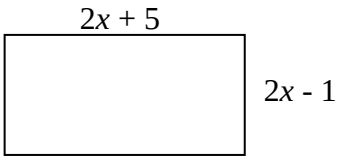
- Describes the information that is represented by an inequality.
- Accepts that a solution which satisfies several conditions can be obtained by solving inequalities.
- Represents the solutions of inequalities on the number line.
- Develops correct relationships through investigation.
- Debates when appropriate and makes the right decisions.

Annex 18.1.1

Instructions for group exploration

Let us solve inequalities

- Focus your attention on the plane figures that belong to your group from the following plane figures

Group	Figure I	Figure II
A		
B		
C		

- Obtain algebraic expressions for the perimeters of Figures I and II.
- Write down an inequality to show that the perimeter of Figure I is greater than the perimeter of Figure II and solve it.
- Graph the solution on a number line.
- Solve the following problem to confirm your findings.
  - An exercise book is sold with a reduction of Rs. 2 on the given price. A book costs more than Rs. 18. Saman who has Rs. 100 can buy a maximum of four books. Take the cost of a book as Rs. *x*, write two inequalities and solve them. What are the integral values that the cost of a book could be?
- Prepare to make a creative presentation of the way you developed the inequalities and found their solutions, at the plenary session.

21. Cyclic Quadrilaterals I

Competency 24 **:** Thinks logically to make decisions based on geometrical concepts related to circles.

Competency Level 24.1 **:** Investigates the quadrilaterals that can be inscribed in a circle.

Activity 24.1 **:** Let us determine the properties of a cyclic quadrilateral.

Time **:** 120 minutes.

Quality Inputs **:**
- The poster included in Annex 24.1.1.
- Four copies of the instruction leaflet on exploration included in Annex 24.1.2.
- Half-sheets, demy papers, pairs of scissors, Bristol boards and pastels.

Learning – Teaching Process:

Step 24.1.1 **:**
- Display the poster in the classroom and inquire from the students about the instances when a straight angle is formed and about cyclic quadrilaterals.
- Lead a discussion and highlight the following facts.
  - That the sum of a pair of adjacent angles on a straight line is 180^o
  - That a quadrilateral with vertices on a circle is a cyclic quadrilateral

(10 minutes)

Step 24.1.2 **:**
- Divide the class into four small groups.
- Distribute the instructions on exploration, half-sheets, pairs of scissors, Bristol boards, demy papers and pastels among the groups.
- Focus the attention of the groups on part I of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(20 minutes)

Step 24.1.3 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That the opposite angles of a cyclic quadrilateral are supplementary
- That the above relationship can be used to solve problems involving cyclic quadrilaterals
- That this relationship can be proved using the theorem that the angle at the center of a circle is twice the angle subtended at the circumference by the same arc

(30 minutes)

Step 24.1.4 **:**
- Organize the groups again.
- Focus the attention of the groups on part II of the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session.

(30 minutes)

Step 24.1.5 **:**
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That quadrilaterals in which opposite angles are supplementary are cyclic quadrilaterals
- That the circle which passes through three vertices of a quadrilateral in which opposite angles are not supplementary, does not pass through the fourth vertex
- That quadrilaterals in which opposite angles are not supplementary are not cyclic quadrilaterals

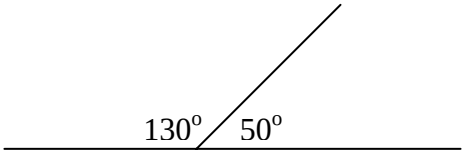
(30 minutes)

Criteria for Assessment and Evaluation:

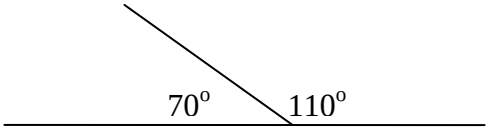
- Accurately expresses the theorems on cyclic quadrilaterals.
- Accepts that quadrilaterals with vertices that lie on a circle have special properties.
- Verifies using geometric constructions that quadrilaterals in which opposite angles are supplementary are cyclic quadrilaterals.
- Confirms facts by using one's creativity.
- Develops the ability to investigate and to debate.

Annex 24.1.1

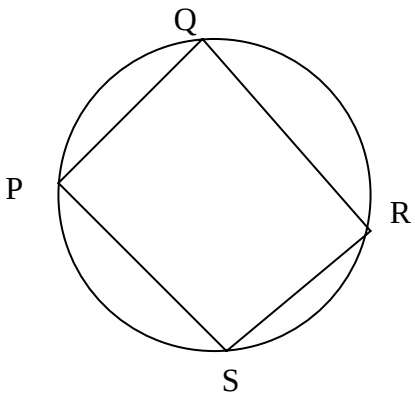
Poster



$130^o + 50^o = 180^o$



$70^o + 110^o = 180^o$



PQRS is a cyclic quadrilateral

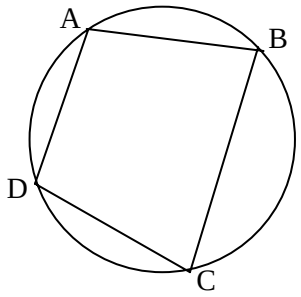
Annex 24.1.2

Instructions for group exploration

Let us determine the properties of a cyclic quadrilateral

Part I

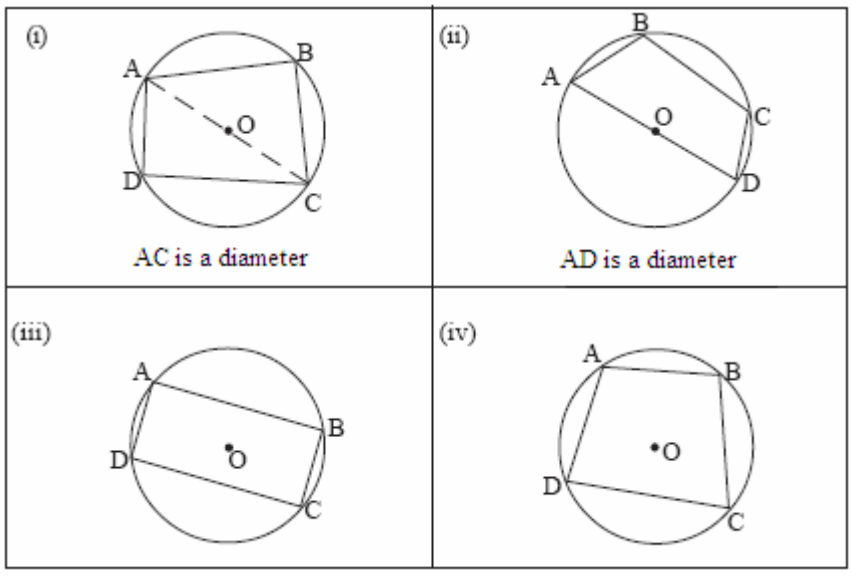
Theorem: ***The opposite angles of a cyclic quadrilateral are supplementary.***



$A\hat{B}C + A\hat{D}C = 180^o$

$B\hat{A}D + B\hat{C}D = 180^o$

- Focus your attention on the figure obtained by your group from the figures given below

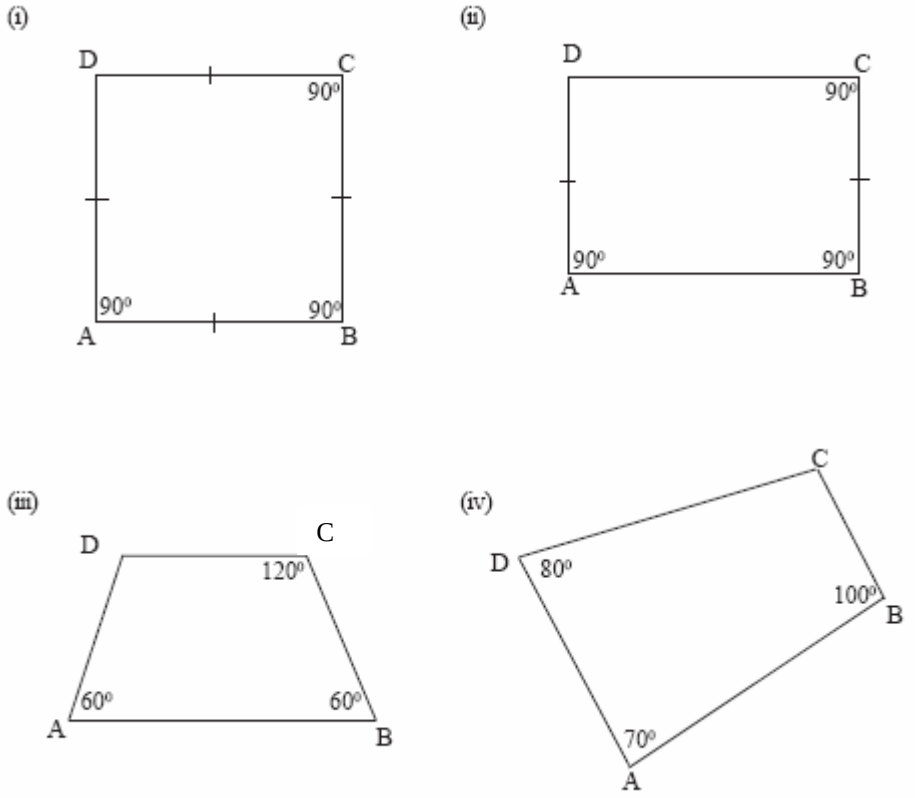


- Draw the figure on a Bristol board such that the radius of the circle is about 5cm.
- Name a pair of opposite angles.
- By either cutting this pair of angles or by some other means check whether they are supplementary.
- Draw a figure of a quadrilateral such that one vertex is not on a circle, and check whether the pairs of opposite angles are supplementary.
- Propose a method to show that the above theorem is true by joining the vertices A and C to the center O and considering the angles $A\hat{B}C$, $A\hat{D}C$ and the angle $A\hat{O}C$.
- Prepare to present your findings to the class.

- **Part II**

Theorem: ***If a pair of opposite angles of a quadrilateral are supplementary, the quadrilateral is a cyclic quadrilateral***

- Focus your attention on the figure received by your group from the figures given below.



- Construct a quadrilateral by taking the sides to be any length you like, and the angles those given in the figure.
- Construct the circle that goes through the points A, B and C and determine whether it goes through the point D too.
- Determine whether the quadrilateral ABCD is a cyclic quadrilateral.
- Draw a quadrilateral such that the sum of a pair of opposite angles is not equal to 180^o and determine whether a circle that passes through the four vertices of the quadrilateral can be drawn.
- Prepare to present your findings to the class.

21. Cyclic Quadrilaterals II

Competency 24 : Thinks logically to make decisions based on geometrical concepts related to circles.

Competency Level 24.2 : Examines the relationship between the exterior and interior angles of a cyclic quadrilateral.

Activity 24.2 : Let us determine the relationship between the exterior and interior angles of a cyclic quadrilateral.

Time : 75 minutes.

Quality Inputs :
- The figure included in Annex 24.2.1.
- Four copies of the instruction leaflet on exploration included in Annex 24.2.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 24.2.1 :
- Present the figure to the class.
- Lead a discussion and highlight the following facts.

  - That an exterior angle can be obtained by producing a side of a polygon
  - That the angle opposite the interior angle which is supplementary to the exterior angle is called the interior opposite angle
  - That if the four vertices of a quadrilateral are concyclic, the quadrilateral is a cyclic-quadrilateral
  - That squares and rectangles can be drawn to be concyclic
  - That opposite angles of cyclic quadrilaterals are supplementary

(15 minutes)

Step 24.2.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(30 minutes)

Step 24.2.3 **:** • Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That the exterior angle formed by producing a side of a cyclic quadrilateral is equal to the interior opposite angle • That the above theorem can be proved by using the theorem that the opposite angles of a cyclic quadrilateral are supplementary • That this relationship can be used to solve problems

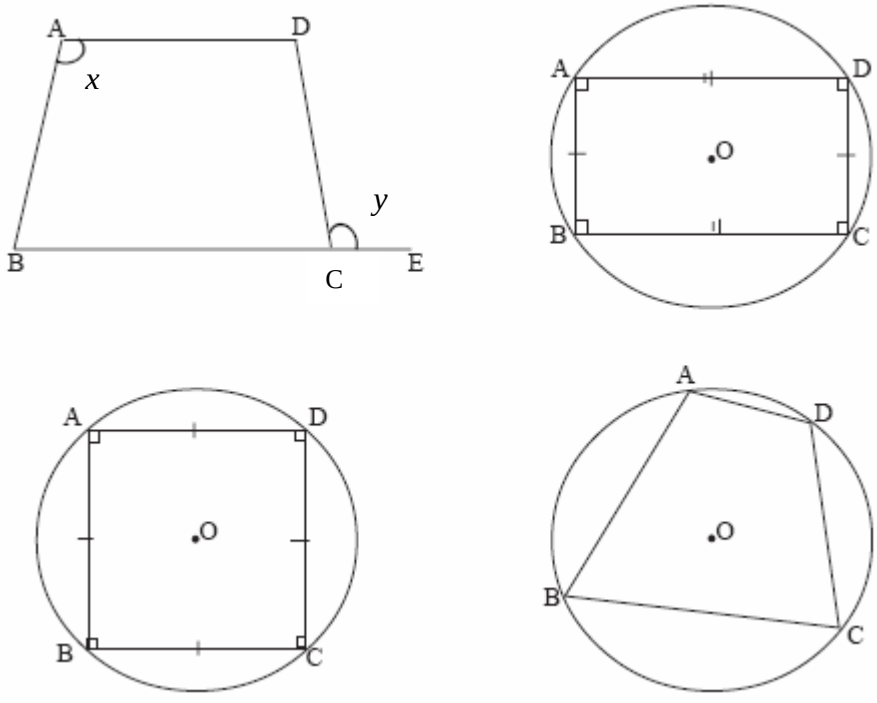
(30 minutes)

Criteria for Assessment and Evaluation:

- Expresses the theorem on the relationship between the exterior angle and the interior opposite angle of a cyclic quadrilateral.
- Accepts that this theorem is valid for any cyclic quadrilateral.
- Uses this relationship to solve problems.
- Confirms an idea by examining it by various methods.
- Approaches the final goal by following a sequence of steps.

Annex 24.2.1

Figure

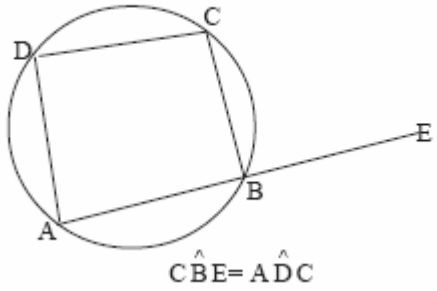


Annex 24.2.2

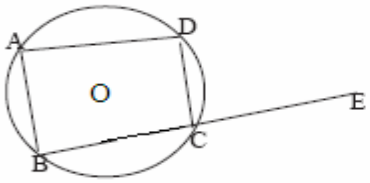
Instructions for group exploration

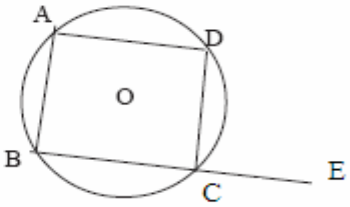
Let us determine the relationship between the exterior and interior angles of a cyclic quadrilateral

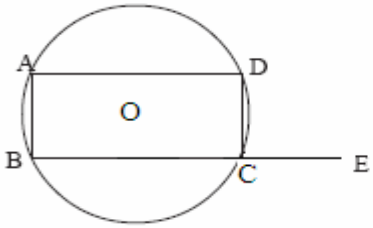
Theorem: ***If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral***

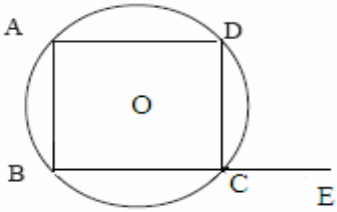


- Focus your attention on the figure that your group receives from the figures given below.









- Draw an enlarged copy of the figure received by your group on the demy paper.
- Examine whether the above theorem is true or false by either cutting the angles and placing them one on top of the other or by some other appropriate means.
- Draw any cyclic quadrilateral you like on the demy paper and discuss how you can prove the relationship between the exterior angle formed by producing a side of the quadrilateral, and the interior opposite angle, using the theorem on cyclic quadrilaterals that you learnt above.
- State the other theorems that you use.
- Creatively prepare to make a presentation at the plenary session.

22. Tangents I

Competency 24 : Thinks logically to make decisions based on geometrical concepts related to circles.

Competency Level 24.3 : Formally confirms the behaviour of angles related to the tangents of a circle.

Activity 24.3 : Let us determine the properties of tangents.

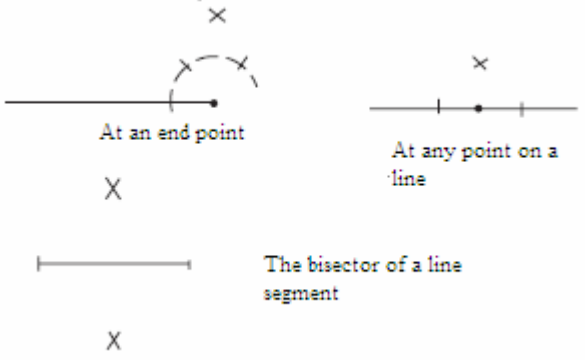
Time : 75 minutes.

Quality Inputs :
- Three copies of the instruction leaflet on exploration included in Annex 24.3.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 24.3.1 :
- Inquire from the students how a perpendicular to a straight line is constructed at various points on the straight line.
- Lead a discussion and highlight the following facts.

- That to construct a perpendicular at the end point of a line segment, an angle of 90^o should be constructed at that point
- That depending on the position of the point at which it is constructed, the method of constructing a perpendicular to a line segment differs



(15 minutes)

Step 24.3.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (30 minutes)

Step 24.3.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That the line drawn through a point on a circle and perpendicular to the radius at that point, is a tangent to the circle
  - That a tangent to a circle meets the circle at exactly one point
  - That the line joining the center of a circle to the point of contact of a tangent to the circle is perpendicular to the tangent
  - That the shortest line segment drawn from a point to a straight line is perpendicular to the straight line

(30 minutes)

Criteria for Assessment and Evaluation:

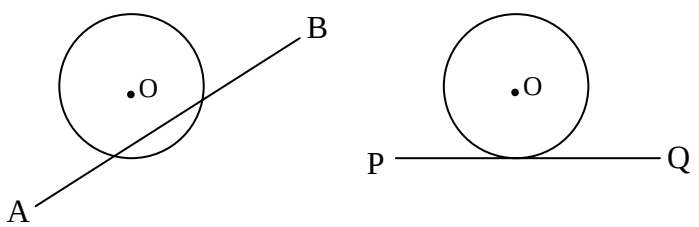
- Expresses the relationship between a tangent and a radius.
- Accepts that when a straight line intersects a circle, it intersects the circle at two points, but that when the straight line is a tangent, it meets the circle at only one point.
- Verifies the theorems related to tangents.
- Presents conclusions through constructions and explorations.
- Logically confirms theories.

Annex 24.3.

Instructions for group exploration

Let us determine the properties of tangents

Study the following carefully



AB is a chord of the circle with centre O
PQ is a tangent to the circle with centre O

Theorem: ***The straight line drawn through a point on a circle and perpendicular to the radius to the point of contact is a tangent to the circle***

Converse: The radius to the point of contact of a tangent is perpendicular to the tangent

- Focus your attention on the section obtained by your group from the sections given below.

Group	A	B	C
Radius	6cm	8cm	10cm

- Draw a circle of the given radius and mark a point P on it.
- Join the centre O and the point P and draw a perpendicular to OP at the point P.
- Discuss within the group the relationship that can be shown to exist between the perpendicular and the circle.
- Can we say that the straight line obtained by extending the perpendicular to the two sides is a tangent?
- If we can say so, discuss the reasons for it.
- Draw another circle of the same radius and name the centre O.
- Using the straight edge draw a straight line segment named AB that touches the circle at one point, and name the point of contact P.
- Join the points O and P and by measuring the angles $O\hat{P}A$ and $O\hat{P}B$ determine whether OP and AB are perpendicular.
- Now mark several points on the tangent on the two sides of P.
- Join these points to the centre O of the circle and measure the distances from these points to the centre.
- What is the shortest distance from a point to a straight line?
- Can you verify the converse of the theorem by using this law?
- Prepare to present your observations and conclusions to the class.

22. Tangents II

Competency 24 : Thinks logically to make decisions based on geometrical concepts related to circles.

Competency Level 24.4 : Examines the properties of tangents drawn from an exterior point to a circle.

Activity 24.4 : Let us learn about tangents.

Time : 75 minutes.

Quality Inputs :
- The poster included in Annex 24.4.1.
- Three copies of the instruction leaflet on exploration included in Annex 24.4.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 24.4.1 :
- Display the poster in front of the class.
- Lead a discussion and highlight the following facts on the congruency of triangles and tangents to a circle.

  - That triangles can be made congruent under 4 cases
  - That the corresponding features of congruent triangles are equal
  - That if a circle and a straight line meet at a unique point, the straight line is a tangent to the circle
  - That a tangent to a circle is perpendicular to the radius to the point of contact

(15 minutes)

Step 24.4.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the groups prepare for a presentation at the plenary session. (40 minutes)

Step 24.4.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

- That only two tangents can be drawn from an exterior point to a circle
- That the two tangents are equal in length
- That the angles subtended at the centre by the tangents are equal
- That the angle between the tangents is bisected by the straight line joining the centre to the exterior point
- That these can be compared by measuring or by using properties of symmetry
- That the relationships connected with the tangents drawn from an exterior point to a circle can be formally proved using the congruency of triangles

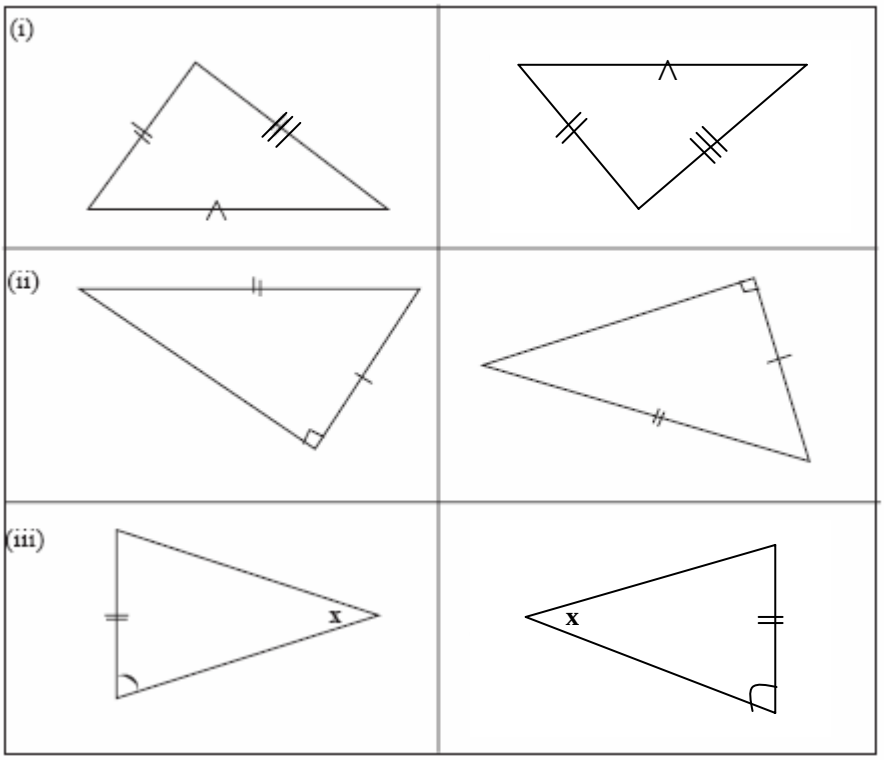
(20 minutes)

Criteria for Assessment and Evaluation:

- Expresses the similarities between the two tangents drawn from an exterior point to a circle.
- Accepts that all the features of a pair of congruent triangles are equal.
- Constructs tangents to a circle at two selected points.
- Is inclined towards writing formal proofs.
- Thinks logically.

Annex 24.4.1

Poster



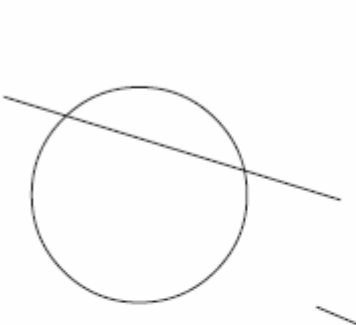


Figure 1

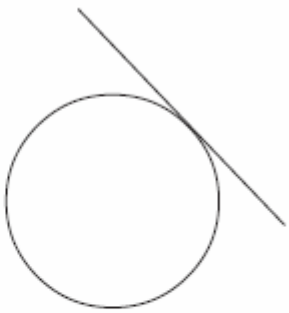


Figure 2

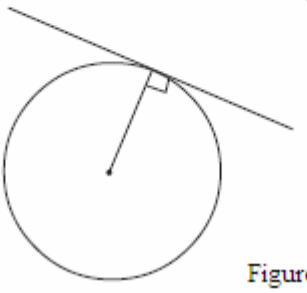


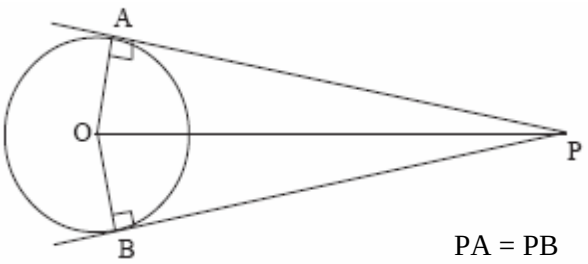
Figure 3

Annex 24.4.2

Instructions for group exploration

Let us learn about tangents

PA and PB are two tangents



PA = PB

$P\hat{O}A = P\hat{O}B$

$O\hat{P}A = O\hat{P}B$

If two tangents are drawn to a circle from an exterior point,

1. the tangents are equal to each other in length
2. the tangents subtend equal angles at the centre
3. the angle between the tangents is bisected by the straight line joining the centre and the exterior point

- Focus your attention on the angle corresponding to a sector obtained by your group from the angles given below.

60^o	90^o	100^o	120^o

- Draw a circle of radius about 4.5cm. Name its centre as O.
- Draw a sector the size of the angle you received.
- Construct two tangents at the points where the radii meet the circle. Extend them until they intersect at a point exterior to the circle. Join the point of intersection to the centre. Propose a method to verify the above theorem using the figure you obtained.
- Draw the figure you obtained again on a piece of paper, name it, and discuss about the data you could use to prove the relationships that you verified in the first exploration. Note this information on the figure.
- Discuss whether the conditions required to make the two triangles congruent are available.
- Use this to determine whether the results of the first exploration can be obtained.
- Prepare to creatively present your findings at the plenary session.

22. Tangents III

Competency 24 : Thinks logically to make decisions based on geometrical concepts related to circles.

Competency Level 24.5 : Examines the relationship between the angle that a tangent to a circle makes with a cord drawn from the point of contact, and the angles in the alternate segments of the circle.

Activity 24.5 : Let us verify the alternate segments theorem.

Time : 75 minutes.

Quality Inputs :
- The chart of figures included in Annex 24.5.1.
- Two copies of the instruction leaflet on exploration included in Annex 24.5.2.
- Protractor.
- Demy papers and pastels.

Learning – Teaching Process:

Step 24.5.1 :
- Present the chart of figures to the class.
- Lead a discussion and highlight the following facts about the tangent, chord and angles in a circle.

• That a line segment which joins two points on a circle is a chord of the circle • That the angle between a tangent and the radius to the point of contact is 90^o • That the angle in a semi-circle is 90^o • That the pair of angles in a right-angled triangle which are not right angles are complementary • That all the angles subtended on the circumference by a segment are equal

(25 minutes)

Step 24.5.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, protractor, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for presentations at the plenary session.

(20 minutes)

Step 24.5.3 : • Provide each small group with the opportunity to present the findings of the group.

- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

  - That the alternate segment can be selected depending on which angle between the chord and the tangent is being considered
  - That the angle in the alternate segment of the circle is the angle subtended on the circumference by the chord
  - That the angle between the chord and the tangent is equal to the angle in the alternate segment
  - That the above theorem can be proved using the diameter drawn through the point of contact

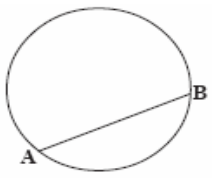
(30 minutes)

Criteria for Assessment and Evaluation:

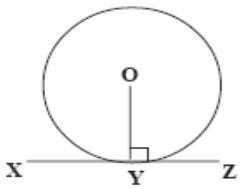
- Expresses what the alternate segment is when an angle is given.
- Accepts that the angle between the tangent and the chord is equal to the angle in the alternate segment.
- Verifies that the angle between the tangent and the chord is equal to the angle in the alternate segment.
- Completes the activity according to the relevant orders.
- Makes decisions logically.

Annex 24.5.1

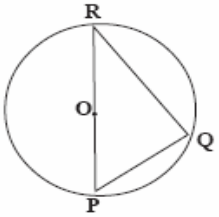
Chart of Figures



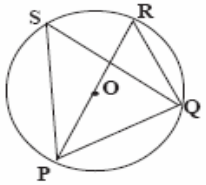
AB is a chord



OY ⊥ XZ



$P\hat{Q}R = 90^{o}$



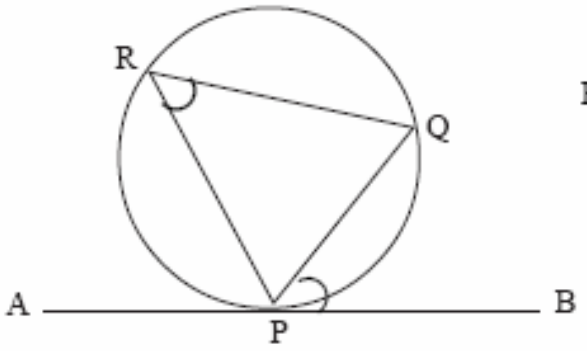
$P\hat{R}Q = P\hat{S}Q$

Annex 24.5.2

Instructions for group exploration

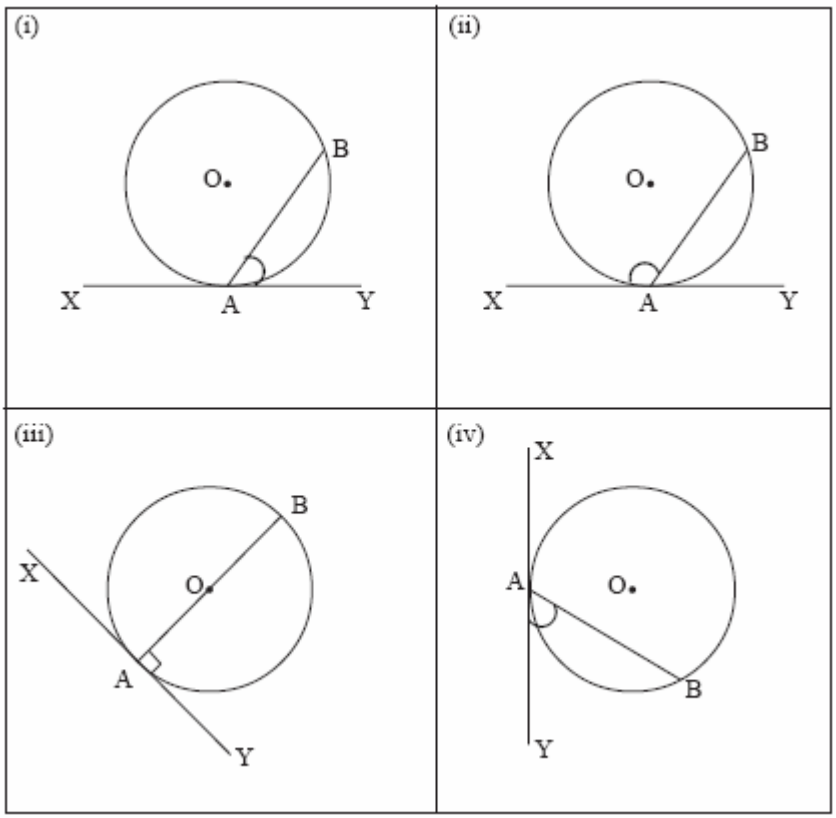
Let us verify the alternate segments theorem

Theorem: ***The angles which a tangent to a circle makes with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle***



$B\hat{P}Q = P\hat{R}Q$

- Focus your attention on the figure obtained by your group from the figures given below.



- Read the section relevant to this lesson from your textbook and identify what an alternate segment related to an angle is.
- By using the chord given in your figure, draw an alternate segment related to the angle given in the figure.
- Verify by a very accurate method that the angle between the chord and the tangent, and the angle in the alternate segment are equal.
- Copy the figure obtained by your group.
- Draw the diameter AC through O.
- Complete the triangle ABC.
- Discuss about a formal method of verifying the theorem using your knowledge on complementary angles.
- Creatively prepare for a presentation of your findings at the plenary session.

23. Constructions I

Competency 27 : Analyzes according to geometric laws, the nature of the locations in the surroundings.

Competency Level 27.1 : Constructs tangents to a circle by using the relationships between the angles related to tangents.

Activity 27.1 : Let us construct tangents to a circle.

Time : 90 minutes.

Quality Inputs :
- The chart of figures included in Annex 27.1.1.
- Copies of the instruction leaflet on exploration included in Annex 27.1.2.
- Boxes of instruments.
- Demy papers and pastels.

Learning – Teaching Process:

Step 27.1.1 :
- Present the chart of figures to the class.
- Lead a discussion and highlight the following facts.

• That only one tangent to a circle can be drawn through a point on the circle • That two tangents can be drawn from an exterior point to a circle • That the tangent at a point on the circle and the radius to the point are perpendicular to each other • That the distance from the exterior point to the two tangent points are equal

(20 minutes)

Step 27.1.2 :
- Divide the class as appropriate into small groups.
- Distribute the instructions on exploration, boxes of instruments, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for presentations at the plenary session.

(30 minutes)

Step 27.1.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That a tangent through a point on a circle can be constructed by drawing a line perpendicular to the radius to the point • That two tangents to a circle can be constructed from an exterior point, by drawing a circle which has as a diameter the line drawn from the exterior point to the centre of the original circle, and then joining the exterior point to the points of intersection of the two circles • That angles of a semi-circle are formed at the intersection points of the given circle and the circle drawn with diameter the line joining the exterior point to the centre of the given circle

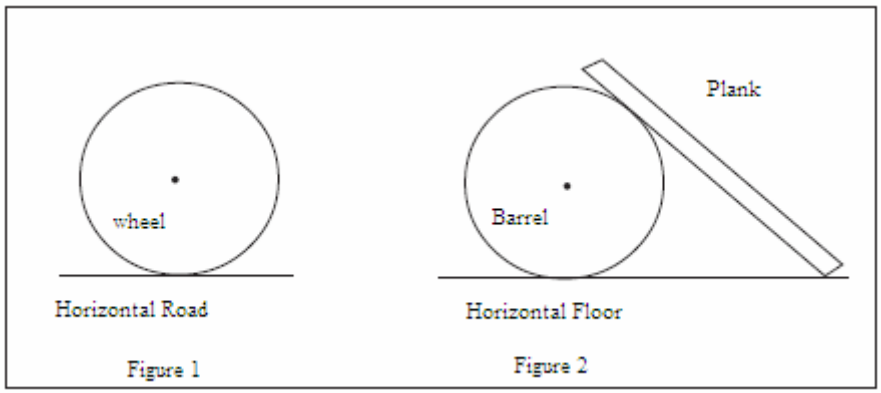
(40 minutes)

Criteria for Assessment and Evaluation:

- Describes how a tangent to a circle is constructed at a given point.
- Accepts that in geometry, a construction can be facilitated by drawing a rough sketch first.
- Constructs tangents to a circle.
- Approaches the solution by removing unsuitable conditions.
- Comes to conclusions by having logical discussions.

Annex 27.1.1

Chart of Figures



- Figure 1 shows the wheel of a moving vehicle on a horizontal road.
- Figure 2 shows an inclined plane used to load items into a lorry.
- Examine how the wheel touches the horizontal road, and how the plank and horizontal floor touch the barrel.

Annex 27.1.2

Instructions for group exploration

Let us construct tangents to a circle

- Focus your attention on the case obtained by your group from the two cases given below.

  Case I : At a point P on a circle
  Case II: From a point P exterior to a circle

- Prepare a rough sketch of a tangent drawn for the case given to your group.
- Construct a tangent with the aid of your rough sketch and your knowledge on the relationships connected with tangents drawn to a circle
- Discuss under which geometric laws your construction valid.
- Prepare for a creative presentation of your findings at the plenary session.

23. Constructions II

Competency 27 **:** Analyzes according to geometric laws, the nature of the locations in the surroundings.

Competency Level 27.2 **:** Investigates the methods of partitioning a straight line segment.

Activity 27.2 **:** Let us divide a straight line segment into equal parts.

Time **:** 70 minutes.

Quality Inputs **:**
- The chart of figures included in Annex 27.2.1.
- Two copies of the instruction leaflet on exploration included in Annex 27.2.2.
- Compasses, set squares and straight edges.
- Demy papers and pastels.

Learning – Teaching Process:

Step 27.2.1 **:**
- Present the chart of figures to the class.
- Lead a discussion and highlight the following facts on parallelism.

• That equal length parts can be marked off on a straight line using a compass • That the gap between two parallel lines is a constant • That when one side of a set square is joined to a straight edge and pushed along the straight edge, the original positions of the other two sides and their final positions are parallel • That a straight line drawn parallel to one side of a triangle and through the mid-point of another side bisects the third side

(20 minutes)

Step 27.2.2 **:**
- Divide the class into three small groups.
- Distribute the instructions on exploration, geometrical instruments, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for presentations at the plenary session.

(20 minutes)

Step 27.2.3 **:**

- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.
- Engage in a review so that the following facts are highlighted.

• That any straight line segment can be divided into equal parts by joining it to a straight line segment which has been divided into the same number of equal parts • That any straight line segment can be divided into equal parts using the converse of the mid-point theorem which states that a straight line segment can be divided into two equal parts

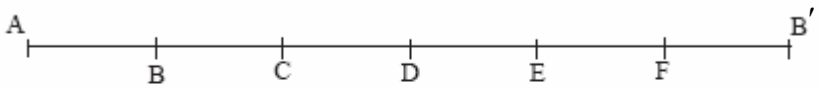
(30 minutes)

Criteria for Assessment and Evaluation:

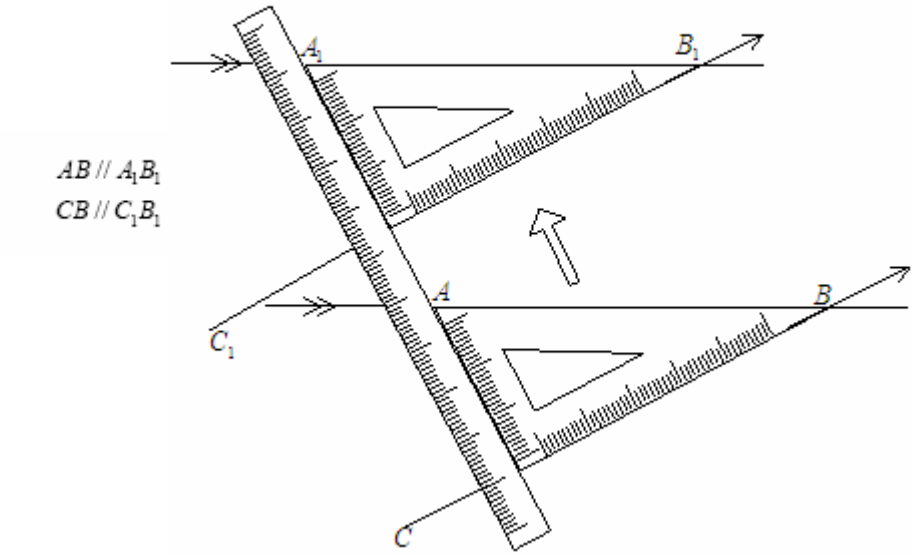
- Describes the method of dividing a straight line segment into several equal parts.
- Accepts that the converse of the mid-point theorem can be used to divide a straight line segment into equal parts.
- Divides a straight line segment into a specified number of equal parts.
- Bases new constructions on previous conclusions.
- Thinks logically.

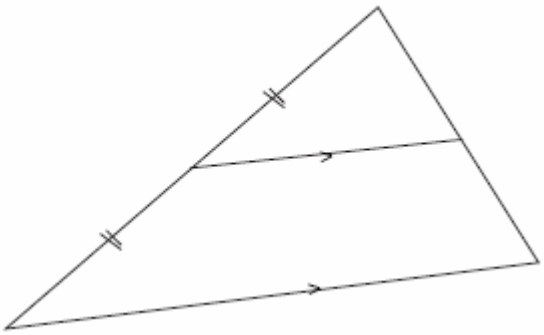
Annex 27.2.1

Chart of Figures



$AB = BC = CD = DE = EF$





The straight line drawn parallel to one side of a triangle and through the mid-point of another side, bisects the third side

Annex 27.2.2

Instructions for group exploration

Let us divide a straight line segment into equal parts

- Focus your attention on the section assigned to your group from the cases given below.

A	Draw a straight line segment of unknown length and divide it into 3 equal parts
B	Draw a straight line segment of unknown length and divide it into 5 equal parts
C	Draw a straight line segment of unknown length and divide it into 7 equal parts

- Consider the converse of the mid-point theorem, that the straight line drawn through the mid-point of one side of a triangle and parallel to another side, bisects the third side.
- Propose a method to divide the straight line segment into the specified number of equal parts, by using a straight line, a section of which is divided equally into the same number of parts.
- Accordingly, do the required construction to divide the straight line segment into the number of equal parts assigned to your group.
- Prepare to present your findings at the plenary session.

24. Sets

Competency 30 : Manipulates the principles related to sets to facilitate daily activities.

Competency Level 30.1 : Identifies the regions of the union and the intersection of sets using Venn diagrams.

Activity 30.1 : Let us represent information using sets.

Time : 75 minutes.

Quality Inputs :
- The chart of Venn diagrams included in Annex 30.1.1.
- Three copies of the instruction leaflet on exploration included in Annex 30.1.2.
- Demy papers and pastels.

Learning – Teaching Process:

Step 30.1.1 :
- Present the chart of Venn diagrams to the class and inquire about their features.
- Lead a discussion and highlight the following facts.

• That two sets with an intersection can be represented by a Venn diagram. • That the intersection, union, complement and universal set can be clearly identified in a Venn diagram • That problems can be solved using the information in a Venn diagram

(15 minutes)

Step 30.1.2 :
- Divide the class into three small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the relevant task to each group.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session. (30 minutes)

Step 30.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.

- Engage in a review so that the following facts are highlighted.

• That before including the data into a Venn diagram of three sets, the correct Venn diagrams should be identified • That a Venn diagram can be drawn easily by examining whether there are elements in the areas of intersection • That the different areas of a Venn diagram should be identified accurately and the related data should be included correctly • That when the given data is included in a Venn diagram, data that is relevant to other information can be generated

(30 minutes)

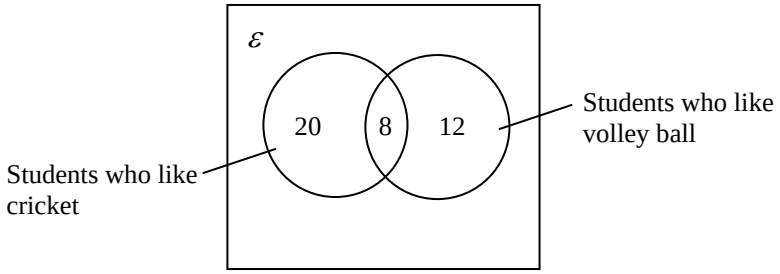
Criteria for Assessment and Evaluation:

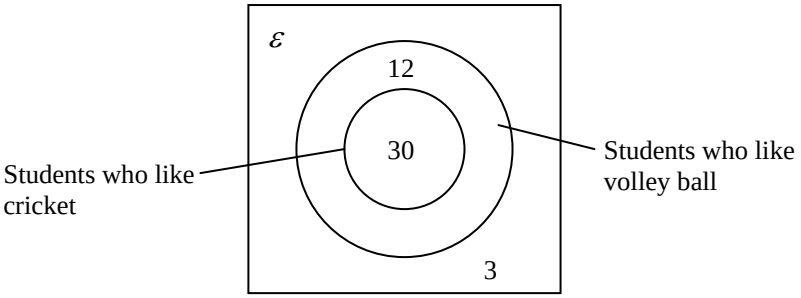
- Describes the areas of a Venn diagram.
- Accepts that it is easier to represent a large amount of information in a Venn diagram, than to write it down.
- Represents information that is given in a Venn diagram.
- Accurately communicates information represented pictorially found in daily life.
- Uses pictorial representations to facilitate problem solving.

Annex 30.1.1

Venn Diagrams

The information on the preference of the students of two classes for volley ball and cricket are represented in the following Venn diagrams



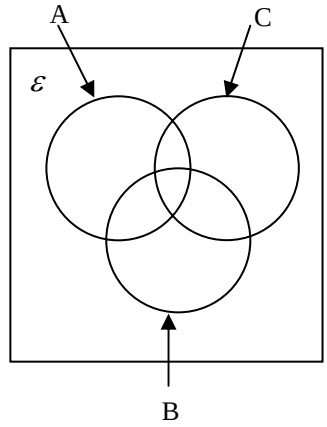
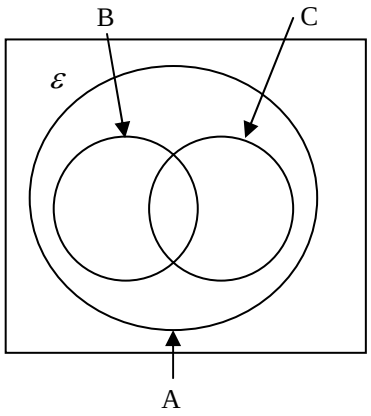
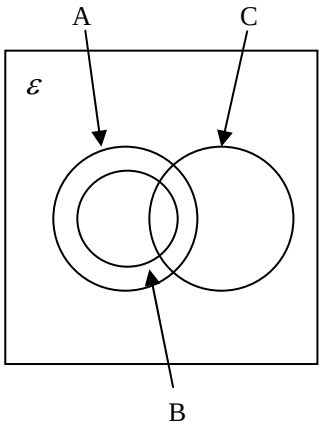


Annex 30.1.2

Instructions for group exploration

Let us represent information using Sets

- Venn diagrams and information regarding the elements of the three sets in the Venn diagrams are given in the three cases below. Focus your attention on the case that your group receives.

Case I	Case II	Case III
		
$n(A) = 14$ $n(B) = 15$ $n(C) = 16$ $n(A \cap B) = 8$ $n(B \cap C) = 6$ $n(A \cap C) = 7$ $n(A \cap B \cap C) = 5$ $n(A \cup B \cup C)' = 3$	$n(A) = 14$ $n(B) = 7$ $n(C) = 8$ $n(A \cap B \cap C) = 5$ $A' = 1$	$n(A) = 14$ $n(B) = 8$ $n(C) = 8$ $n(A \cap C) = 3$ $n(A \cap B \cap C) = 1$ $n(A \cup B \cup C)' = 3$

- Include the given information in the Venn diagram.
- Find $n(\varepsilon)$ by means of it.
- Shade the areas that are common to only two sets.
- Find an event in daily life that can be represented by the Venn diagram that was received by your group and include the numbers related to the event in the relevant areas.
- Prepare for a creative presentation of your findings at the plenary session.

25. Probability

Competency 31 : Analyzes the likelihood of an event occurring to predict future events.

Competency Level 31.1 : Interprets daily occurrences using the various methods of representing the likelihood of an event occurring.

Activity 31.1 : Let us represent the sample space of an event by a tree diagram.

Time : 60 minutes.

Quality Inputs :
- Four copies of the instruction leaflet on exploration included in Annex 31.1.1.
- Demy papers and pastels.

Learning – Teaching Process:

Step 31.1.1 :
- Inquire from the students about the sample space of an experiment such as throwing two die together, and representing it in a grid.
- Lead a discussion and highlight the following facts.

  - That the probability of the event A occurring within a sample space S is given by $P(A) = \frac{n(A)}{n(S)}$
  - That for two mutually exclusive events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(20 minutes)

Step 31.1.2 :
- Divide the class into four small groups.
- Distribute the instructions on exploration, demy papers and pastels among the groups.
- Focus the attention of the groups on the instructions on exploration and assign the same task to two groups each.
- Engage the small groups in exploration.
- Let the small groups prepare for a presentation at the plenary session.

(20 minutes)

Step 31.1.3 :
- Provide each small group with the opportunity to present the findings of the group.
- Give the presenters themselves the first opportunity to elaborate on the presentation.
- Seek for constructive comments from the other groups.

- Engage in a review so that the following facts are highlighted.

- That in representing events by a tree diagram, the number of branches equals the number of possible events
- That the sum of the probabilities of the branches for each situation equals 1
- That to find the probability of a compound event, the probabilities along the branches should be multiplied
- That if a certain outcome is generated in more than one way, the probability of the outcome is the sum of the probabilities down the relevant branches
- That the sum of the probabilities of the events obtained by first multiplying along the branches and then adding down the branches equals 1

(20 minutes)

Criteria for Assessment and Evaluation:

- Describes how tree diagrams are used to solve problems related to probabilities.
- Accepts that it is easy to calculate the probability of a compound event by using a tree diagram.
- Solves problems related to probabilities using tree diagrams.
- Obtains solutions using models.
- Uses methods of simplifying complex events.

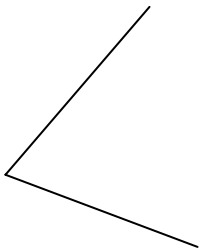
Annex 31.1.1

Instructions for group exploration

Let us represent the sample space of an event by a tree diagram

- Focus your attention on the chart given below.

A box of pens contains 4 blue pens and 3 red pens. To represent the events of a pen taken at random being a blue pen/ not a blue pen, a student had drawn the following figure.



- With the aid of the textbook discuss how the above diagram can be used to represent the sample space of the events of a pen being blue/not blue.
- Accordingly, note the probabilities of the events on the diagram.
- Focus your attention on the event received by your group from the events given below

Event 1	Event 2
The pen drawn at random is not replaced.	The pen drawn at random is replaced.

- Propose a method to include the sample space of the next set of events in the above diagram, when two pens are drawn at random according to the method received by your group.
- Note the probabilities in the diagram and find the probability of getting 2 blue pens.
- Create a problem similar to the given example, and include the sample space for two stages in a diagram similar to the above.
- Calculate the probabilities of the events related to each and every branch and write them down.
- Propose a name for the diagram by focusing your attention on the properties of the diagram and on the probabilities.
- Prepare for a presentation at the plenary session.

Assessment and Evaluation

Introduction

Assessment and evaluation can be defined as two interconnected programmes that could be conveniently implemented in the classroom in order to identify the levels of competence achieved by the students and to establish that the students have actualized the learning outcomes expected through the learning-teaching process. If the assessment is implemented properly, it is not difficult for all the students studying in the class to acquire a competency at least proximate to the relevant skill. On the other hand, what is expected from evaluation is the identification of the levels of competence that the students have achieved.

Teachers involved in assessment can provide their students with two types of guidance. These are commonly called "feedback" and "feed forward". It is the task of the teacher to provide feedback to students to overcome their learning difficulties, when the weaknesses and inabilities of students are discovered, and to provide feed forward to further improve their skills when their abilities and strengths are discovered.

For the success of the learning-teaching process, it is necessary that students discover which competencies they have been able to actualize and the relevant levels. It is expected therefore that the teacher will judge the competency levels achieved by the students through the evaluation process, and communicate their progress to the students themselves, to their parents as well as other relevant parties.

This curriculum has a student centered, competency-based, activity oriented approach. In the transformation role of the teacher, 'learning through action for a meaningful life' is the core.

An attempt has been made in this curriculum which is implemented through a pre-determined activity continuum, to integrate assessment and evaluation with learning and teaching. The teacher will be able to assess the students when they are involved in exploration within groups under the second step of each activity, and then to evaluate them when they present their findings and subject the same to elaboration. The teacher is expected to move among the students engaged in exploration, observe the tasks they are involved in, help them to solve in the classroom itself any problems they happen to encounter and provide them with facilities and guidance.

Five common criteria are suggested to facilitate the task of assessment and evaluation. Out of these criteria, the first three are based on knowledge, attitudes and skills that combine to develop each competency. The final two criteria support students in mastering two skills that are important in life. The teacher should make an effort to identify the five behavioral changes related to these criteria within the classroom itself while the students are active, strengthen them under assessment and make judgments regarding the level of achievement under evaluation.

The learning-teaching process can be broadened through the improvement of the assessment and evaluation programme. To broaden the learning teaching process in this manner, the

activities in the activity continuum should first be divided into several clusters. Next, learning aids that will enhance student learning and are based on the syllabus content related to each activity cluster should be identified. Preparing the instrument which will broaden the learning teaching process is the next step. The instrument should be based on the selected learning aids and should include instructions for the teacher as well as for the students. It is expected that the teacher will introduce the instrument to the students at the beginning of the activity cluster. Below is a list of learning aids that can be used to prepare instruments that broaden the learning teaching process.

Concept maps
Wall newspapers
Quizzes
Question and answer books
Portfolios
Exhibitions
Debates
Panel discussions
Seminars
Impromptu speeches
Role play
Presentation of literature reviews
Field books/ nature diaries
Practical tests

The third part of this teacher's instructional manual has been planned in order to introduce instruments to broaden the learning teaching process. By assessing and evaluating in a two-fold manner, between and through the activities, the learning teaching process is further extended with students involving themselves in learning with interest and understanding.

Instruments for the extension of the learning – teaching process

Instrument - 01

01. Time of evaluation : 1st term

02. The competency levels covered : 1.1**,** 1.2, 6.1, 6.2 and 6.3

03. Relevant subject content :
- Set Notations (set of natural numbers, set of integers, set of rational numbers, set of irrational numbers, set of real numbers)
- Simplification (surds, entire surds)
- Rationalizing the denominator (only the form $\frac{a}{\sqrt{b}}$)
- Integral indices
- Rational indices
- Logarithmic laws (for powers and roots)
- Expressions with powers and roots
- Equations with powers and roots (without using logarithmic tables)
- Logarithms of numbers less than 1
- Expressions including numbers less than 1
  - Multiplication
  - Division
- Verifying results using the calculator

04. Nature of the instrument :
- A Questions and Answers Competition.

05. Aims of the instrument :
- Creates problems on real numbers, indices and logarithms.
- Meaningfully analyses the solutions to questions.
- Identifies the relationships between the various sections of the subject.
- Works with a sense of team spirit and commitment when creating problems and solutions.
- Applies the knowledge on real numbers and indices to facilitate day to day life.

06. Instructions to implement the instrument

For the teacher :
- Introduce this instrument to the class at the beginning of the lesson on real numbers.
- Divide the class into groups such that the maximum number of students in a group is four.
- Inform the students about the relevant subject content and criteria for allocating marks.

- Instruct the students to prepare a question paper consisting of 20 problems that cover the given subject content, and a solution sheet for it.
- Decide on the total mark for the paper and inform the students to also prepare a detailed marking scheme.
- A week after activity 6.3 is concluded and after the accuracy of the students' work is examined, exchange the question papers between two groups and give the groups sufficient time to answer the questions
- After the questions have been answered, give the solutions to the groups that prepared the questions to be evaluated and for marks to be allocated.

For the students :

- Prepare a question paper with 20 problems on the subject content given by the teacher, within a week after the lesson is completed.
- Prepare a detailed marking scheme taking into consideration the solution sheet and the total mark allocated by the teacher.
- Exchange the question paper with the question paper prepared by the group that your teacher names, and provide answers to their questions.
- Exchange the solution sheets and mark them.

07. Method of allocating marks

Criteria :

- Prepares the question paper relevant to the group.
- Presents the answer sheet and the marking scheme.
- Successfully provides solutions to another group's question paper.
- Completes the assigned work in the allotted time.
- Works in corporation and with team spirit.

Marks range :

Very Good	04
Good	03
Average	02
Should Improve	01

Instrument - 02

01. Time of evaluation : 1st term
02. The competency levels covered : 8.1, 10.1
03. Relevant subject content :
- Surface areas of solids
  - Pyramid with a square base
  - Cone
  - Sphere
- Volume of solids
  - Pyramid with a square base
  - Cone
  - Sphere

04. Nature of the instrument :
- A Creative Activity

05. Aims of the instrument :
- Uses formulae accurately to find the surface areas and volumes of solids.
- Discusses the changes in a cone which depend on the sector the cone is formed from.
- Explains how the surface area and volume of a sphere changes depending on the radius.
- Makes creations by combining solids.

06. Instructions to implement the instrument

For the teacher :
- Introduce this instrument before commencing activity 8.1.
- Divide the students into groups such that the maximum number in a group is four.
- Inform the students that they should hand in their completed work within a week after the end of activity 10.1.
- Provide the students with the required instructions and the criteria for the allocation of marks during the activity.
- Allocate marks according to the given criteria.

For the students :
- Study the following instructions carefully in groups and engage in the activity.
- Inquire and learn from the teacher any other facts that are required.
- Cut out several circles of radius 7cm.
- Cut out from these circles, sectors with angles of 60^o, 90^o, 120^o, 150^o, 180^o at the centre.
- Calculate the length of the arc of each of these sectors
- If cones are made from these sectors, determine the radii of the bases of these cones using the lengths of the arcs.
- Calculate the altitude.
- Note the information you obtained in the following table

Angle	Length of the arc	Radius of the base		Altitude	
		By calculating	By measuring	By calculating	By measuring
60^o					
90^o					
120^o					
150^o					
180^o					

- Create cones using the above sectors.
- Now measure the radius of the base of the cone and the altitude and fill in the blanks of the table.
- Compare the values you got by calculating and by measuring and see if they are the same.
- Explain the relationship between the radius and the surface area as well as between the radius and the volume of a sphere when the radius of the sphere is increased in the ratio 1:2:3 etc . (Find the surface area and volume of several spheres of radii 7cm, 14cm, 21cm ....)
- Create a suitable trophy using a pyramid with a square base and a sphere. (Can divide into parts according to the need)
- Calculate the volume of the trophy. If the mass of $1cm^3$ of the trophy is 50g, find the total mass of the trophy.

07. Method of allocating marks

Criteria :

- Calculates using the correct formulae, the change in the radius of the base and the altitude of a cone when the angle of the sector used to form the cone varies.
- Compares by creating cones, how the base radius and the altitude change depending on the size of the angle of the sector, and verifies the accuracy of the answers
- Represents clearly the relationship between the change in the surface area and volume with the change in the radius.
- Makes an attractive product in the allotted time.
- Makes new creations with the knowledge gained on solids.

Marks range :

Very Good	04
Good	03
Average	02
Should Improve	01

Instrument - 03

01. Time of evaluation : 1st term
02. The competency levels covered : 14.1, 16.1 and 23.1
03. Relevant subject content :
- Binomial Expressions
- Algebraic Fractions
- The areas of plane figures between parallel lines

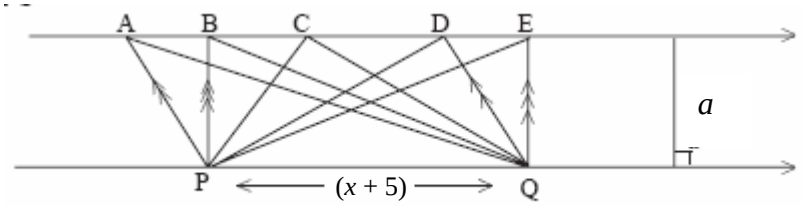
04. Nature of the instrument :
- An investigative study

05. Aims of the instrument :
- Identifies the various shapes in a compound plane figure.
- Develops relationships by being attentive to the areas of triangles and parallelograms between a pair of parallel lines.
- Obtains the product of two binomial expressions.
- Inquires into the numerical values that the unknown term can take, when the length of a side of a rectilinear plane figure is expressed in terms of an unknown.

06. Instructions to implement the instrument

For the teacher :
- Divide the class into groups such that the maximum in a group is four.
- Introduce this instrument before the commencement of activity 14.1.
- Give each group a copy of the students' instruction leaflet.
- Obtain the students' outcomes within a week after the completion of activity 23.1 and lead a discussion on them.
- Allocate marks according to the given criteria by considering the students' work while they are engaged in the activity as well as by considering the final outcome.

For the students :



- Study the given figure carefully and name the triangles and parallelograms in the figure.
- Develop as many relationships as possible between the areas of the triangles and parallelograms you identified above and write them down.
- If the length of the line segment PQ is given by $(x + 5)$ and the perpendicular distance between the two straight lines PQ and AE is a, complete the following table for the different values of a given in terms of x.

Value of a	The area of ADPQ (algebraically)
1 $(x + 5)$	
2 $(x + 3)$	
3 $(x – 2)$	
4 $(x – 5)$	

- Discuss the ideas about the location of the above plane figure when x = -5. (Hint: Calculate the length of PQ)
- If the area of ADPQ in each of the above 4 cases is 24 square units, determine the value of x for each case.
- Using the values obtained for x, find the length of the line segment PQ for each case.

07. Method of allocating marks

Criteria **:**

- Develops the relationships between the areas of the triangles and parallelograms in the figure.
- Obtains the product of two binomial expressions.
- Expresses ideas about the location of the plane figure, based on the value of x.
- Finds the length of the line segment PQ by solving quadratic equations.
- Works within the group such that corporation and unity are preserved.

Marks Range **:**

Very Good	04
Good	03
Average	02
Should Improve	01

Instrument - 04

01. Time of evaluation : 2nd term
02. The competency levels covered : 5.1, 5.2 and 5.3
03. Relevant subject content :
- Loan installments and hire purchases
- Compound interest
- Joint businesses and shares

04. Nature of the instrument :
- A Question and Answer Competition

05. Aims of the instrument :
- Selects the organization that gives the best deal for purchases where payment is made in installments.
- Discusses the advantages and disadvantages of buying items outright and on hire purchase.
- Calculates the interest obtained for deposits and the dividend obtained for shares.
- When investing money, compares and studies the most beneficial schemes.

06. Instructions to implement the instrument:

For the teacher :
- Inform the students about this instrument before commencing activity 5.1
- Divide the class into groups such that the maximum number in a group is four.
- Give each group a copy of the students' instruction leaflet.
- Inquire into the students' progress and provide the necessary advice/instructions.
- Collect the students' final outcomes within a week of the completion of activity 5.3 and lead a discussion on them.
- Allocate marks according to the given criteria.

For the students :
- Get together as a group, study the given instructions carefully and engage in the given activity.
- The table given below provides information on how a computer can be bought from three different organizations.

Organization	Cost of item for an outright payment	When bought on hire purchase		
		Initial payment	Number of Installments	Value of an installment (Rs.)
A	38000.00	8000.00	15	2480.00
B	38000.00	2000.00	12	3624.00
C	38000.00	8000.00	12	2955.00

- According to the above table, if the computer is bought on hire purchase, which organization is it most advantageous to buy from?
- Which organization has the minimum annual interest rate? (Explain your answer by giving reasons)
- Discuss the advantages and disadvantages of buying the item outright and buying on hire purchase.

Several methods of investing money are given below from which a person with Rs. 50000.00 to invest can select to invest by.

Method 1 – Depositing for year, in a bank which pays a compound interest rate of 12% per annum. (Interest is calculated semi-annually)

Method 2 – Purchasing Rs. 10 shares for Rs. 8 of a company that pays an annual dividend of 15%.

Method 3 – Purchasing Rs. 15 shares for Rs. 18 of a company that pays an annual dividend of 16%.

- Write giving reasons, which method is the most advantageous to invest the money in for a year.
- Discuss the various methods of investing money and the advantages and disadvantages of each method.

07. Method of allocating marks

Criteria :

- Accurately manipulates the data to select a suitable organization to make a purchase from.
- Calculates the profits made by investing money in various ways.
- Makes decisions on the most suitable methods of investment, by making comparisons.
- Works in corporation and with commitment as groups.
- Presents the final outcomes meaningfully and clearly.

Marks Range :

Very Good	04
Good	03
Average	02
Should Improve	01

Instrument - 05

01. Time of evaluation : 2nd term

02. The competency levels covered : 23.3, 23.4 and 23.5

03. Relevant subject content :
- The relationship between the sides of a triangle and parallelism
- Equi-angularity of two triangles, the related theorem and its converse
- The mid-point theorem and its converse

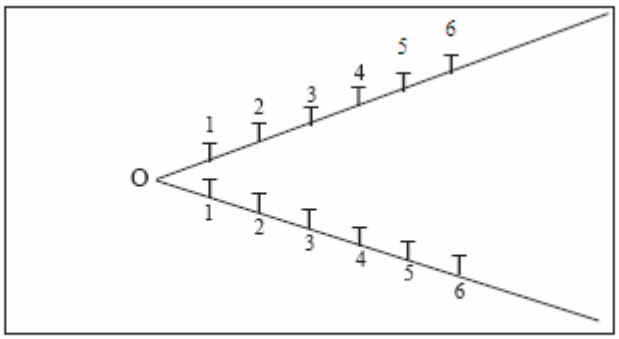
04. Nature of the instrument :
- A Creative Activity

05. Aims of the instrument :
- Discovers the various relationships that can be applied to a given plane figure.
- Confirms the geometric relationships that have been learnt, through activities.
- Is inclined to focus investigative attention on the various locations in the surroundings.

06. Instructions to implement the instrument

For the teacher :
- Provide information to the students about this instrument and the criteria for allocating marks before starting activity 23.3.
- Give instructions to fix nails 1cm apart from each other on a plank and number them as shown in the figure.
- During the activity, by making inquires, direct the students to deduce geometric theorems using the prepared plank, rubber bands, straight edge, protractor, thread etc.
- Provide the students with the opportunity to present their findings.

Figure



Plank

For the students :
- Prepare the plank with nails as instructed by the teacher.
- Work according to the instructions given by the teacher and write down a list of geometric relationships that can be deduced, by creating rectilinear plane figures on the prepared plank.
- Prepare to explain to others by making demonstrations, the relationships you obtained using the plank.
- Propose methods to further improve the instrument to confirm facts in mathematics.

07. Method of allocating marks

Criteria :
- Prepares the final outcomes accurately, clearly and with cleanliness.
- Lists the geometric relationships that can be deduced using the instrument.
- Uses the instrument to explain the geometric relationships to others.
- Works creatively and in corporation within a group.
- Puts forward proposals to improve the instrument.

Marks Range :

Very Good	04
Good	03
Average	02
Should Improve	01

Instrument - 06

01. Time of evaluation : 2nd term

02. The competency levels covered : 28.1, 28.2 and 28.3

03. Relevant subject content :
- Representation and interpretation of data.
  - Histogram
  - Frequency Polygon
  - Cumulative Frequency Curve
    - Quartiles
    - Inter-quartile Range

04. Nature of the instrument :
- An Explorative Study

05. Aims of the instrument :
- Gathers information accurately.
- Represents the obtained information in a frequency table with appropriate class intervals.
- Draws the relevant histogram.
- Draws the frequency polygon by means of the histogram or some other method.
- Gathers data from the surroundings and represents it in a cumulative frequency curve.
- Finds the quartiles and the inter-quartile range using the cumulative frequency curve

06. Instructions to implement the instrument:

For the teacher :
- Inform the students about this instrument before commencing activity 28.1.
- Divide the class into groups of 4.
- Inform the students about the nature of the instrument and the criteria for allocating marks.
- Give instructions to the groups to select 5 students each, from each of the grades 1, 2, 3, 4, 5. (Selection of the students can be changed according to the teacher's preference)
- At the conclusion of the lesson, inform the students of the time allocated for the activity and provide them the opportunity to present their findings.
- Instruct them to start collecting the relevant data from the surroundings and to engage in the activity from the commencement of activity 28.1.
- Examine the students' activities during decided times.
- Allocate marks according to the given criteria

For the students :

- Measure and note down the weights of the students you selected, according to the teacher's instructions.
- Enter the information that you gathered in a frequency table with appropriate class intervals.
- Represent this information in a histogram.
- Draw the frequency polygon, using the histogram or by some other method.
- Prepare another frequency table with class intervals, using information collected from the surroundings.
- Using the mid-value and the cumulative frequency column, represent the data you obtained by a cumulative frequency curve.
- Find the quartiles and the inter-quartile range by using the cumulative frequency curve.
- Propose a method to obtain the most accurate value for the median by using the cumulative frequency curve.
- Obtain the median by the method you propose.
- Prepare to present your completed work when the time allocated for this activity is over.

07. Method of allocating marks

Criteria :

- Gathers accurate data and prepares frequency tables with class intervals.
- Draws a histogram.
- Draws the frequency polygon using the histogram or by some other method.
- Obtains the quartiles and the inter-quartile range using the cumulative frequency curve.

Marks Range :

Very Good	04
Good	03
Average	02
Should Improve	01

Theorems under the Theme of Geometry from Grade 6 to Grade 11

1. **Theorem** : The sum of the adjacent angles formed by a straight line meeting another straight line, equals two right angles.

2. Theorem : If two straight lines cut one another, the vertically opposite angles are equal. (Proof necessary)

3. Theorem : If a transversal cuts a pair of parallel lines, then
i. the corresponding angles are equal to each other.
ii. the alternate angles are equal to each other.
iii. the sum of a pair of allied angles equals two right angles

4. Theorem : When two straight lines are cut by a transversal, if

i. a pair of corresponding angles are equal to each other or
ii. the alternate angles are equal to each other or
iii. the sum of a pair of allied angles equals two right angles,

then the two straight lines are parallel to each other.

The four instances of congruency of triangles

5. Theorem : If the two sides and the included angle of one triangle are equal to the two sides and the included angle of another triangle, the two triangles are congruent. (s.a.s.)

6. Theorem : If two angles and one side of one triangle are equal to two angles and the corresponding side of another triangle, then the two triangles are congruent. (a.a.s)

7. Theorem : If the three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent. (s.s.s)

8. Theorem : If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and one side of another right-angled triangle, then the two triangles are congruent. (hyp. s)

9. Theorem : If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. (Proof necessary)

10. Theorem : The sum of the interior angles of a triangle is 180^o.

11. Theorem : The sum of the interior angles of an n-sided polygon equals (2n – 4) right angles.
: The sum of the exterior angles of an n-sided polygon equals 360^o.

12. Theorem : If two sides of a triangle are equal, the angles opposite those sides are equal. (Proof necessary)

13. Theorem : If two angles of a triangle are equal, then the sides which are opposite to those angles are equal.

14. Theorem : In a parallelogram, opposite sides are equal, opposite angles are equal, and each diagonal bisects the parallelogram. (Proof necessary)

15. Theorem : The diagonals of a parallelogram bisect each other.

16. Theorem : If the opposite sides of a quadrilateral are equal, it is a parallelogram.

17. Theorem : If the opposite angles of a quadrilateral are equal, it is a parallelogram.

18. Theorem : If the diagonals of a quadrilateral bisect each other, it is a parallelogram.

19. Theorem : If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram.

20. Theorem : The straight line segment joining the mid-points of two sides of a triangle is parallel to the third side, and equal to half of it. (Proof necessary)

21. Theorem : The straight line through the mid-point of one side of a triangle and parallel to another side, bisects the third side.

22. Theorem : Parallelograms on the same base and between the same parallel lines are equal in area.

23. Theorem : If a parallelogram and a triangle stand on the same base and between the same parallel lines, the area of the triangle is half that of the parallelogram.

24. Theorem : Triangles on the same base and between the same parallel lines are equal in area.

25. Theorem : The areas of triangles of equal altitude are proportional to each other as their bases.

26. Theorem : In a right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described on the other two sides.

27. Theorem : A straight line drawn parallel to one side of a triangle cuts the other two sides, or those sides produced, proportionally.

28. Theorem : If two sides of a triangle are divided in the same ratio by a straight line, then the straight line is parallel to the third side.

29. Theorem : If two triangles are equi-angular to one another, their corresponding sides are proportional.

30. Theorem : If the sides of two triangles are proportional, the triangles are equi-angular.

31: Theorem : The radius through the mid-point of a chord is perpendicular to the chord. (Proof necessary)

32. Theorem : The perpendicular from the centre of a circle to a chord, bisects the chord.

33. Theorem : The angle at the centre of a circle is equal to twice the angle subtended on the circumference by the same arc. (Proof necessary)

34. Theorem : Angles in the same segment of a circle are equal.

35. Theorem : The angle in a semi-circle is a right-angle.

36. Theorem : The opposite angles of a cyclic quadrilateral are supplementary. (Proof necessary)

37. Theorem : If a pair of opposite angles of a quadrilateral are supplementary, its vertices are con-cyclic.

38. Theorem : If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral.

39. Theorem : The straight line drawn through a point on a circle and perpendicular to the radius through the point of contact, is a tangent to the circle.

40. Theorem : The tangent at any point on a circle is perpendicular to the radius drawn through the point of contact.

41. Theorem : If two tangents are drawn to a circle from an exterior point, then
i. the two tangents are equal to each other in length.
ii. the tangents subtend equal angles at the centre.
iii. the angle between the tangents is bisected by the straight line joining the exterior point to the centre.

42. Theorem : The angles which a tangent to a circle makes with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle.

Competencies identified for Mathematics for Grades 6 to 11

Competency – 1

Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.

Competency – 2

Makes decisions for future requirements by investigating the various relationships in number patterns.

Competency – 3

Manipulates units and parts of units under the mathematical operations to easily fulfill the requirements of day to day life.

Competency – 4

Uses ratios to facilitate day to day activities.

Competency - 5

Makes successful transactions in the modern world of commerce by using percentages.

Competency – 6

Easily solves mathematical problems in day to day life by using logarithms and calculators.

Competency – 7

Carries out daily tasks effectively by investigating the various methods of finding the perimeter.

Competency – 8

Makes use of a limited space in an optimal manner by investigating the area.

Competency – 9

Fulfills daily requirements by working with an awareness of mass.

Competency – 10

Gets the maximum out of space by working critically with respect to volume.

Competency – 11

Fulfills daily needs by working critically with the knowledge of liquid measures.

Competency – 12

Fulfills the needs of the world of work by time management.

Competency – 13

Uses scale drawings in practical situations by exploring various methods.

Competency – 14

Simplifies algebraic expressions by systematically exploring various methods.

Competency – 15

Factorizes algebraic expressions by systematically exploring various methods.

Competency - 16

Solves problems encountered in day to day life by exploring the various methods of simplifying algebraic fractions

Competency – 17

Manipulates the methods of solving equations to fulfill the needs of day to day life.

Competency – 18

Analyzes the relationships between various quantities related to real-life problems.

Competency – 19

Solves problems encountered in day to day life by exploring the methods by which formulae can be applied.

Competency – 20

Easily communicates the mutual relationships that exist between two variables by exploring various methods.

Competency – 21

Makes decisions by investigating the relationships between various angles.

Competency – 22

Creates new models by exploring various solids.

Competency – 23

Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency – 24

Thinks logically to make decisions based on geometrical concepts related to circles.

Competency – 25

Studies the beauty of the environment by exploring the properties of various shapes.

Competency – 26

Investigates the methods of organizing various geometric shapes and uses them to

enhance beauty.

Competency – 27

Analyzes according to geometric laws, the nature of the locations in the surroundings.

Competency – 28

Facilitates daily work by investigating the various methods of representing data.

Competency – 29

Makes predictions by analyzing data by various methods to facilitate daily activities.

Competency – 30

Manipulates the principles related to sets to facilitate daily activities.

Competency – 31

Analyzes the likelihood of an event occurring to predict future events.